Advanced Particle Physics 2

Strong Interactions and Weak Interactions

L10 – Neutrino Interactions

(http://dpnc.unige.ch/~bravar/PPA2/L10)

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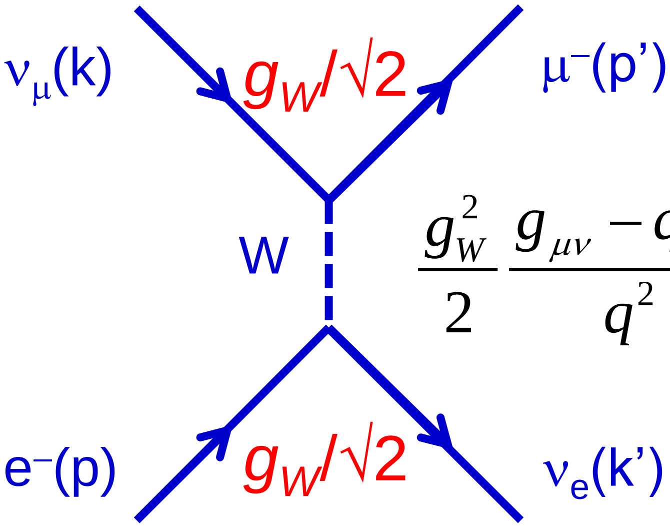
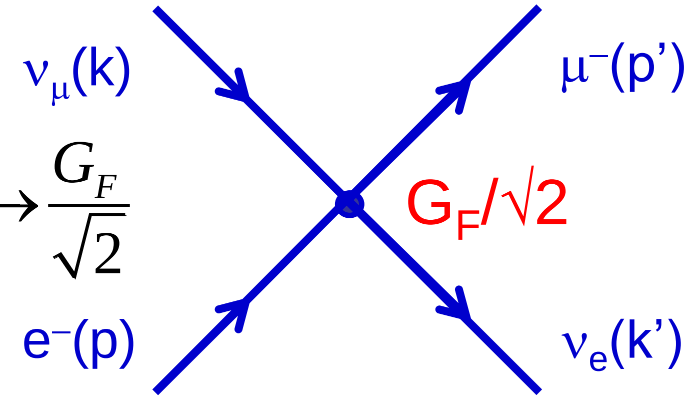
Charged Current $\nu_\mu\, e^- \to \mu^-\, \nu_e$ Scattering
(inverse μ decay)



t – channel

$$\frac{g_W^2}{2}\,\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \xrightarrow{q^2 \to 0} \frac{G_F}{\sqrt{2}}$$



effective 4 fermion interaction (point like a la Fermi)

$$M = \frac{g_W}{\sqrt{2}}\left(\bar{u}(p')\gamma^\mu \frac{1}{2}(1-\gamma^5)u(k)\right)\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_w^2}\frac{g_W}{\sqrt{2}}\left(\bar{u}(k')\gamma^\nu \frac{1}{2}(1-\gamma^5)u(p)\right)$$

$$M = \frac{4G_F}{\sqrt{2}}\Big(\bar{u}_L(p')\gamma^\mu u_L(k)\Big)\Big(\bar{u}_L(k')\gamma_\mu u_L(p)\Big)$$

$$\begin{cases} u_L = \frac{1}{2}(1-\gamma^5)u \\ \bar{u}_L = \bar{u}\,\frac{1}{2}(1+\gamma^5) \end{cases}$$

$J_\mu^\dagger$ – charge lowering weak current $\qquad J^\mu$ – charge raising weak current

Invariant Amplitude $\nu_\mu e^- \rightarrow \mu^- \nu_e$

Invariant amplitude

$$M = \frac{4G_F}{\sqrt{2}}\left[\bar{u}(p')\gamma^\mu \frac{1}{2}(1-\gamma^5)u(k)\right]\left[\bar{u}(k')\gamma_\mu \frac{1}{2}(1-\gamma^5)u(p)\right]$$

Spin averaged |invariant amplitude|^2

$$\left\langle |M|^2 \right\rangle = \frac{1}{2s+1}\sum_{\text{spins}}|M|^2$$

$$= \frac{1}{2}\frac{16G_F^2}{2}\sum_{s,t}\left[\bar{u}^s(p')\gamma^\mu \frac{1}{2}(1-\gamma^5)u^t(k)\right]\left[\bar{u}^t(k)\frac{1}{2}(1+\gamma^5)\gamma^\nu u^s(p')\right]\times$$

$$\sum_{s',t'}\left[\bar{u}^{s'}(k')\gamma_\mu \frac{1}{2}(1-\gamma^5)u^{t'}(p)\right]\left[\bar{u}^{t'}(p)\frac{1}{2}(1+\gamma^5)\gamma_\nu u^{s'}(k')\right]$$

Apply Casimir's trick (Γ_i – 4 x 4 matrix)

$$\sum_{spins}\left[\bar{u}(a)\Gamma_1 u(b)\right]\left[\bar{u}(a)\Gamma_2 u(b)\right]^* = \mathrm{Tr}\left[\Gamma_1(\not{p}_b + m_b)\ \gamma^0\Gamma_2\gamma^0(\not{p}_a + m_a)\right]$$

Finally …

$$\left\langle |M|^2 \right\rangle = \frac{1}{2}\frac{G_F^2}{2}\mathrm{Tr}\left\{\not{k}\gamma^\mu(1-\gamma^5)(\not{p}' + m_e)\gamma^\nu(1-\gamma^5)\not{k}'\right\}\times\mathrm{Tr}\left\{\gamma_\mu(1-\gamma^5)\not{k}\gamma_\nu(1-\gamma^5)(\not{p}' + m_\mu)\right\}$$

$$= 64G_F^2(k\cdot p)(k'\cdot p')$$ (neglecting m_e and m_μ)

compare with muon decay (L8) → "inverse" muon decay

$$\left\langle |M|^2 \right\rangle = \frac{1}{2} \sum_{\substack{\text{initial}\\ \text{final}\\ \text{spins}}} |M|^2 = \cdots\cdots\cdots = 64 G_F^2 (k\cdot p)(p'\cdot k')$$

$$\begin{cases} m_\mu = 0 \qquad m_e = 0 \\ s = (p+k)^2 = (p'+k')^2 \approx 2p\cdot k = 2p'\cdot k' \end{cases}$$

$$= 16 G_F^2 s^2$$ no angular dependence in |amplitude|^2

$$\mathrm{d}\sigma(\nu e) = \frac{1}{F}\left\langle |M|^2 \right\rangle \mathrm{d}Q \leftarrow \begin{cases} F = 4|\vec{p}_i|\sqrt{s} \\ \mathrm{d}Q = \dfrac{1}{4\pi^2}\dfrac{|\vec{p}_f|}{4\sqrt{s}}\mathrm{d}\Omega \end{cases} \qquad s = 2p\cdot k = 2E_\nu m_e$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\nu e) = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}\left\langle |M|^2 \right\rangle$$

$p_f = p_i$ in the limit m_e , $m_\mu = 0$

$$= \frac{G_F^2 s}{4\pi^2} \xrightarrow{\int_\Omega = 4\pi} \sigma = \frac{G_F^2}{\pi} s \xrightarrow{LAB} \frac{G_F^2}{\pi} 2E_\nu m_e$$

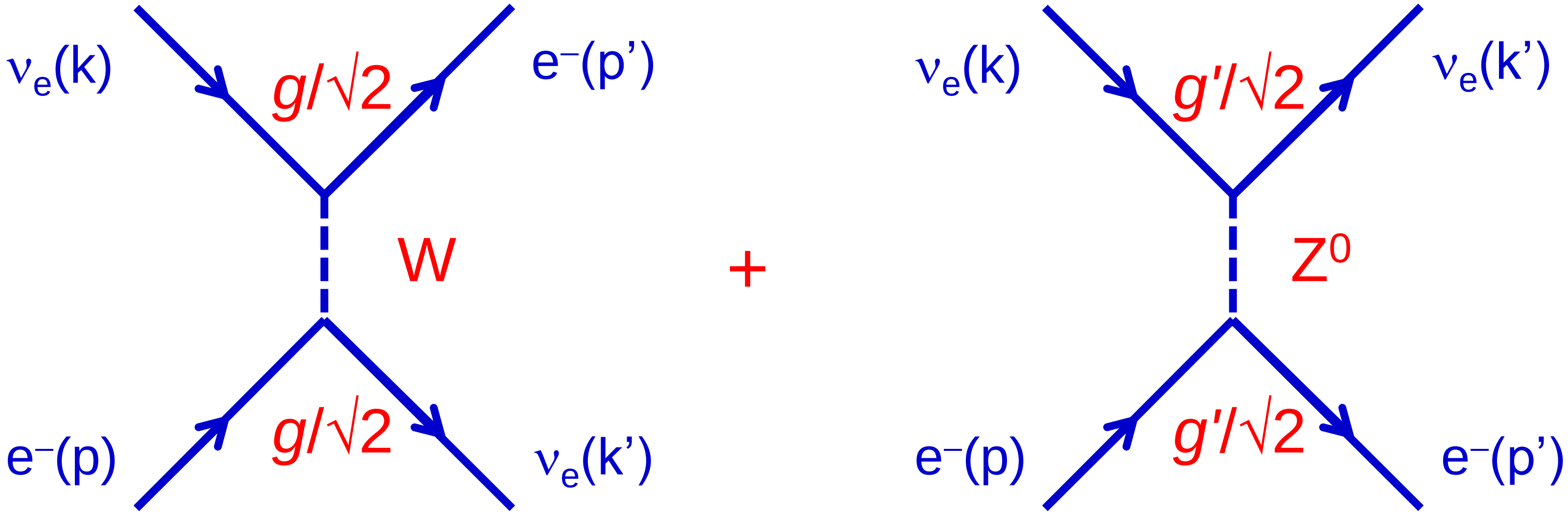
$$\sigma_{LAB} = 1.7\times 10^{-41} E_\nu [\mathrm{GeV}]\ \mathrm{cm}^2$$

extremely small
study ν scattering off nuclei
much larger cross sections

angular distribution shows directly the *V – A* structure
isotropic! (angular momentum conservation)
σ grows linearly with E_ν indefinitely! (no propagator)
(in L7 we arrived at this conclusion using dimensional arguments only)

$\nu_e\, e^- \rightarrow e^-\, \nu_e$ Scattering

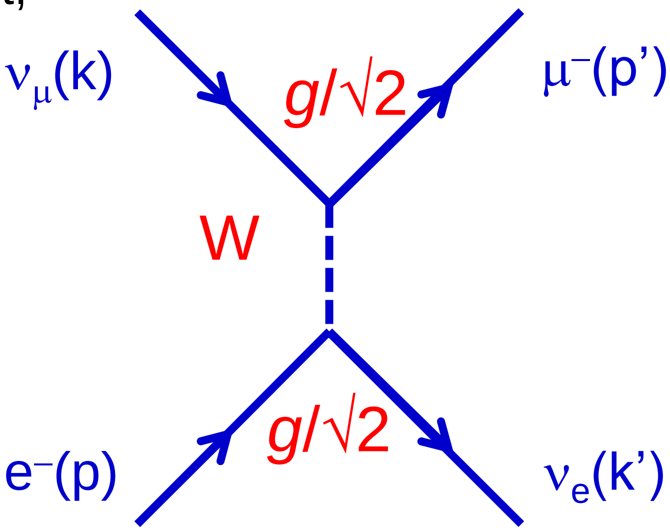
To be complete, have to consider also neutral currents …



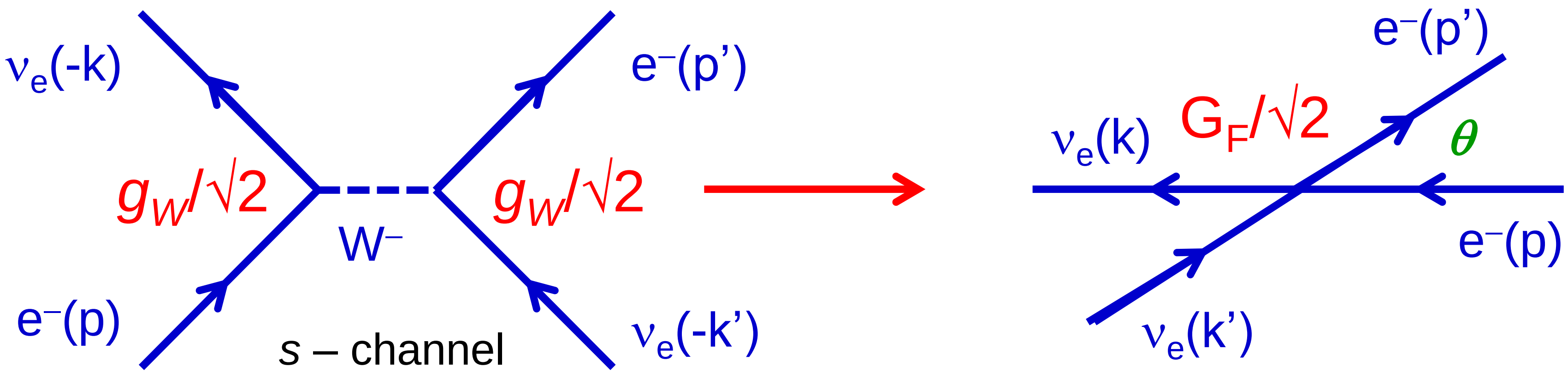
The neutral current interferes with the charged current; to obtain the amplitude for $M(\nu_e e \rightarrow \nu_e e)$ we have to add both diagrams:

$M = M^{CC}(\nu_e e \rightarrow e \nu_e) + M^{NC}(\nu_e e \rightarrow \nu_e e)$

In practice high energy neutrino beams are obtained from charged pion decays: $\pi^+ \rightarrow \mu^+ \nu_\mu$. Neutrino beams are composed mainly of muon neutrinos (99%) and $\nu_\mu e \rightarrow \mu \nu_e$ scattering proceeds via charged current interactions only.



Charged Current $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$ Scattering



obtained by crossing CC t-channel νe → νe scattering amplitude

$$M(k,p,k',p') = M(-k',p,-k,p') \quad s \leftrightarrow t$$

$$\langle |M|^2 \rangle = \cdots\cdots\cdots = 16 G_F^2 t^2 \qquad t \approx -\frac{1}{2} s(1+\cos\vartheta)$$

$$= 4 G_F^2 s^2 (1+\cos\vartheta)^2$$

max for $\theta = 0$, min for $\theta = 180^0$

$$\frac{d\sigma}{d\Omega}(\bar{\nu}e) = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}\langle |M|^2 \rangle = \frac{G_F^2}{16\pi^2} s(1+\cos\vartheta)^2$$

angular dependence !

$$\sigma(\bar{\nu}e) = \frac{1}{3}\frac{G_F^2}{\pi} s \xrightarrow{LAB} \frac{1}{3}\frac{G_F^2}{\pi} 2E_\nu m_e$$

$$\sigma(\bar{\nu}e) = \frac{1}{3}\sigma(\nu e)$$

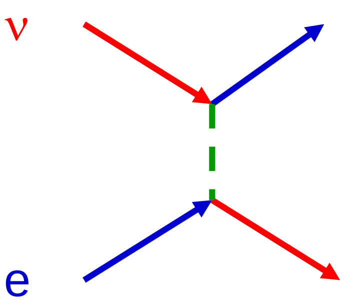
Helicity Considerations

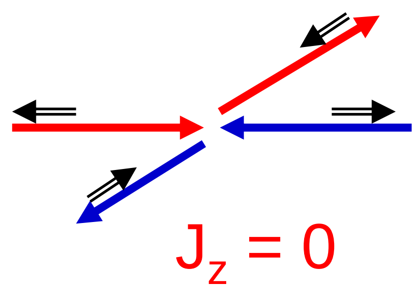
weak interactions couple to
left-handed ($\sim\lambda = -1$) particles and
right-handed ($\sim\lambda = +1$) anti-particles

$$\frac{d\sigma(\nu e)}{dy} = \frac{G_F^2}{\pi} s$$

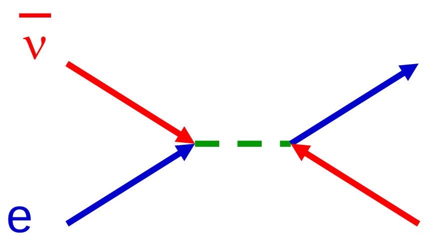
$$\frac{d\sigma(\bar{\nu} e)}{dy} = \frac{G_F^2}{\pi} s (1-y)^2$$

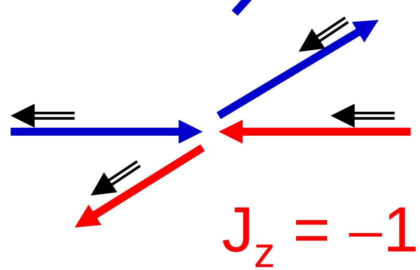
$$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{1}{3}$$





same handedness
no net spin along beam direction
$\rightarrow$ isotropic distribution: 1





Recall e$\mu \rightarrow$ eμ scattering

$$\frac{d\sigma(e\mu \rightarrow e\mu)}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} s\left[1 + (1-y)^2\right]$$

isotropic, parallel helicities J = 0
antiparallel helicities J = 1

opposite handedness
net spin J_z = +1 or -1 (only 1 possibility out of 3)

$\rightarrow$ scattering amplitude: $y = 1 - \frac{p \cdot k'}{p \cdot k} \cong \frac{1}{2}(1 - \cos\vartheta)$

y measures the energy transfer at the interaction vertex ($\approx E_W / E_\nu$)

Neutral Currents

1973 experimental birth of Standard Model

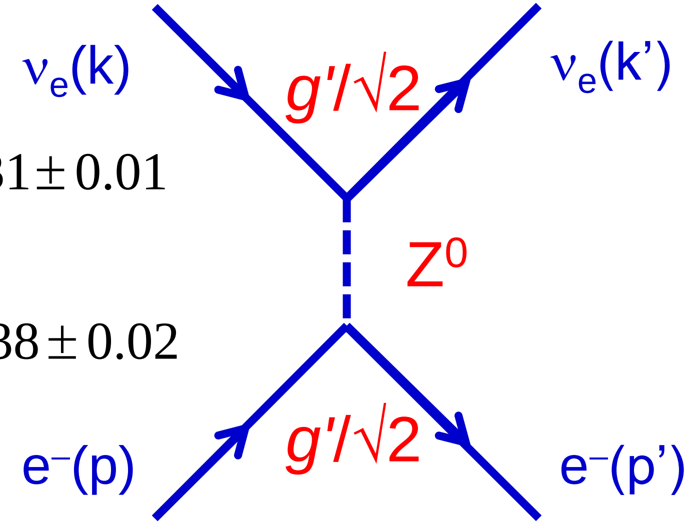
$\bar{\nu}_\mu e \to \bar{\nu}_\mu e$

$\nu_\mu N \to \nu_\mu X$

$\bar{\nu}_\mu N \to \bar{\nu}_\mu X$

$$R_\nu = \frac{\sigma^{NC}(\nu)}{\sigma^{CC}(\nu)} = \frac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu^- X)} \approx 0.31 \pm 0.01$$

$$R_{\bar{\nu}} = \frac{\sigma^{NC}(\bar{\nu})}{\sigma^{CC}(\bar{\nu})} = \frac{\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\bar{\nu}_\mu N \to \mu^+ X)} \approx 0.38 \pm 0.02$$



First evidence of a weak neutral current

NC anticipated by Glashow in 1961
Until then no weak neutral current effects have been observed
Note: no flavor change at the vertex, NC conserve flavor!

Very stringent limits on (flavor changing) neutral currents by the absence of decays

$$K^0 \to \mu^+\mu^- \qquad BR = 7\times10^{-9}$$

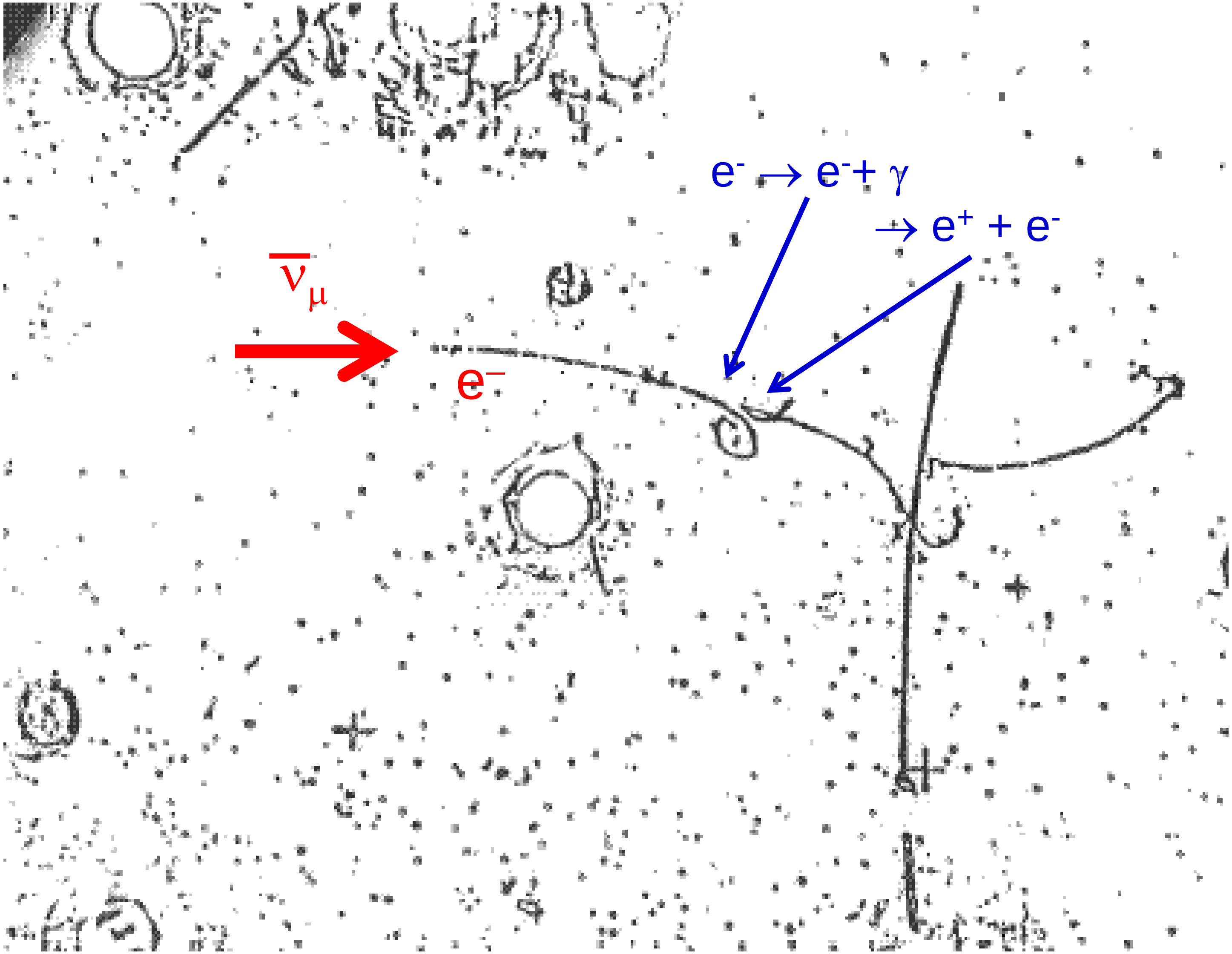
$$K^+ \to \pi^+\mu^+\mu^- \qquad BR < 4\times10^{-11}$$

These small (non-zero!) branching ratios explained well by SM (GIM mechanism), also:

$$B_s^0 \to \mu^+\mu^- \qquad BR = 3\times10^{-9}$$

However in νe, νq scattering NC events are as abundant as CC events,
difficult to detect isolated electron, study on nuclear targets.

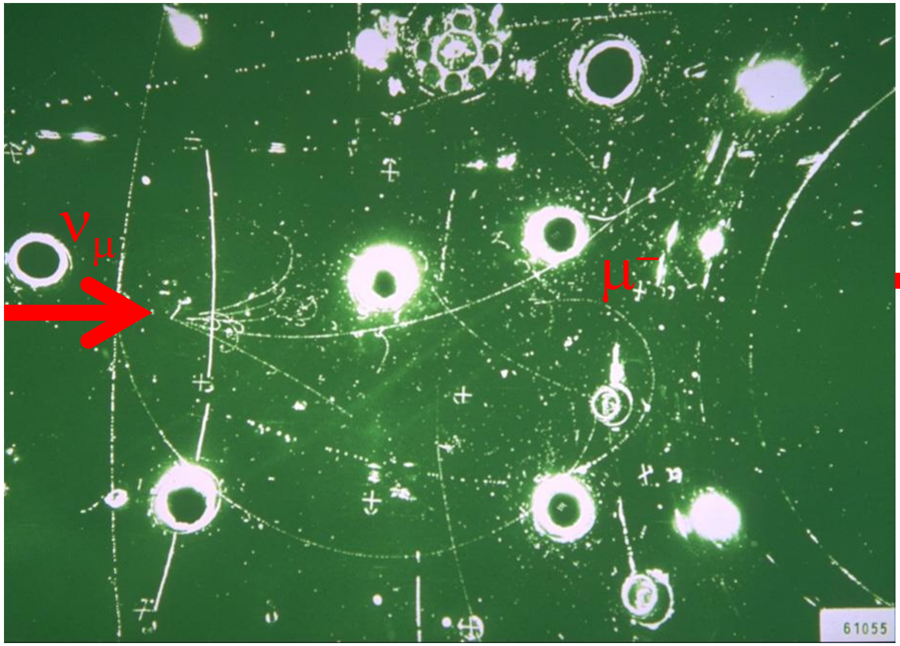
Neutral Currents: $\bar{\nu}_\mu$ e- $\rightarrow \bar{\nu}_\mu$ e- Scattering



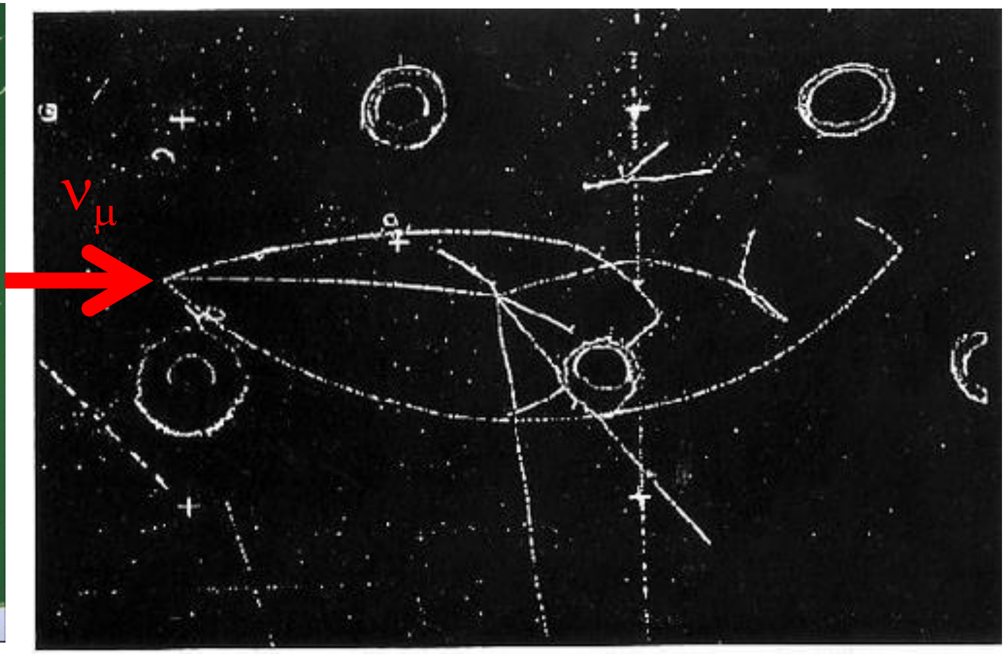
CC and NC – νN Scattering

CC: $\nu_\mu N \to \mu^- + X$

NC: $\nu_\mu N \to \nu_\mu + X$



one lepton (μ^-) detected
all other particles identified as hadrons



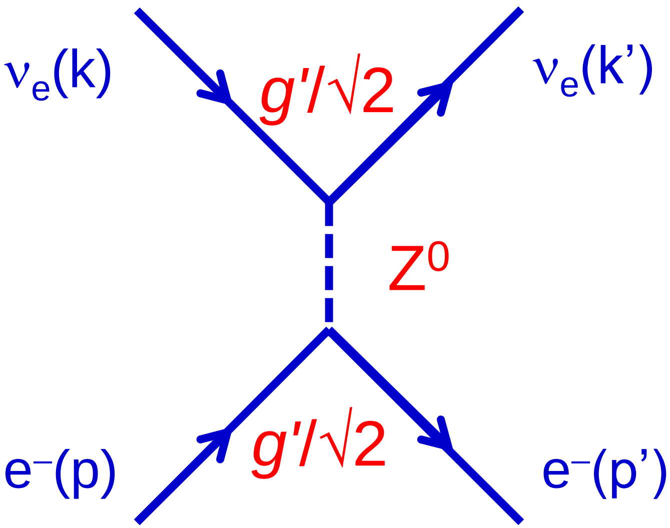
all particles identified as hadrons
no leptons detected!

$$R_\nu = \frac{\sigma^{NC}(\nu)}{\sigma^{CC}(\nu)} = \frac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu^- X)} \approx 0.31 \pm 0.01$$

$$R_{\bar\nu} = \frac{\sigma^{NC}(\bar\nu)}{\sigma^{CC}(\bar\nu)} = \frac{\sigma(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma(\bar\nu_\mu N \to \mu^+ X)} \approx 0.38 \pm 0.02$$

almost as abundant as CC

NC Scattering Amplitude



develop in analogy to CC at low $q^2 \ll M_Z^2$

a priori:

i) not necessarily pure V – A, what structure?

ii) can have right handed components (not for ν)
try $c_V V - c_A A$ (c_V and c_A from experiment)

iii) new coupling g', new massive neutral boson

iv) no flavor change at the interaction vertex $\delta_{ff'}$

$$M^{NC} = \frac{g'}{\sqrt{2}}\left(\bar{u}_e \gamma^{\mu} \tfrac{1}{2}(c_V^e - c_A^e \gamma^5) u_e\right) \frac{g_{\mu\nu} - q_\mu q_\nu / M_Z^2}{q^2 - M_Z^2} \frac{g'}{\sqrt{2}}\left(\bar{u}_\nu \gamma_\mu \tfrac{1}{2}(c_V^\nu - c_A^\nu \gamma^5) u_\nu\right)$$

effective 4-fermion theory as for CC with new coupling constant $G_{NC} / \sqrt{2} = g'^2 / 8 M_Z^2$
and $c_V^\nu = c_A^\nu = 1/2$ (neutrinos are left-handed) [in a V + A theory $c_V^\nu = -c_A^\nu = 1/2$]

$$M^{NC} = \frac{4G_{NC}}{\sqrt{2}} 2\left(\bar{u}_e \gamma^{\mu} \tfrac{1}{2}(c_V^e - c_A^e \gamma^5) u_e\right) \frac{1}{2}\left(\bar{u}_\nu \gamma_\mu \tfrac{1}{2}(1 - \gamma^5) u_\nu\right)$$

$(J^{NC})^\mu$ (e) $\qquad$ $(J^{NC})_\mu$ (ν)

neutrino neutral current $$J^{NC}{}_{\mu}(\nu) = \frac{1}{2}\left[\bar{u}_{(\nu)}\,\gamma_{\mu}\,\frac{1}{2}(1-\gamma^5)\,u_{(\nu)}\right]$$

electron neutral current $$J^{NC\mu}(e) = \left[\bar{u}_{(e)}\,\gamma^{\mu}\,\frac{1}{2}(c_V^e - c_A^e\gamma^5)\,u_{(e)}\right]$$

"point-like" interaction of two neutral currents $(J^{NC})^{\mu}$ (e) and $(J^{NC})_{\mu}$ (ν)

$$M^{NC} = \frac{4G_F}{\sqrt{2}}2\rho J_{\mu}^{NC}(e)J^{NC\mu}(\nu) \qquad \rho = \frac{G_{NC}}{G_F} \approx 1.010 \pm 0.015 = 1 \quad \text{(SM)}$$

ρ determines the relative strength of NC to CC, in the SM $\rho = 1$

In the SM all c_V^i and c_A^i are given in terms of one parameter,

the electroweak mixing Weinberg angle θ_W

$$\tan\vartheta_W = g'/g \qquad e = g\cdot\sin\vartheta_W = g'\cdot\cos\vartheta_W$$

θ_W measures the relative strength of CC and NC couplings with $\rho = \frac{M_W^2}{M_Z^2\cos^2\vartheta_W} = 1$

$c_V^e = -1/2 + 2\sin^2\theta_W \qquad c_A^e = -1/2$

(all this will be developed in L11)

In summary, we have a basis for calculating NC amplitudes.
From now on, assume $\rho = 1$ and $G_{NC} = G_F$. The only unknowns are c_V^e and c_A^e.

NC $\nu_e e^- \to \nu_e e^-$ Cross Sections

To start, let's consider $\nu_\mu e^-$ or $\nu_\tau e^-$ scattering (no CC channel!). The NC amplitude is

$$M^{NC}(\nu_\mu e^- \to \nu_\mu e^-) = \frac{4G_F}{\sqrt{2}} 2\rho \left(\bar{u}_e \gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e \gamma^5) u_e \right) \frac{1}{2} \left(\bar{u}_\nu \gamma_\mu \tfrac{1}{2}(1-\gamma^5) u_\nu \right)$$

Using the CC current results

$$\frac{d\sigma(\nu_e e^- \to e^- \nu_e)}{dy} = \frac{G_F^2}{\pi} s \quad \text{and} \quad \frac{d\sigma(\bar{\nu}_e e^- \to \bar{\nu}_e e^-)}{dy} = \frac{G_F^2}{\pi} s(1-y)^2$$

("left-handed") ("right-handed")

we obtain directly

$$\frac{d\sigma^{NC}(\nu_\mu e^- \to \nu_\mu e^-)}{dy} = \frac{G_F^2 s}{4\pi} \Big[\underbrace{(c_V^e + c_A^e)^2}_{c_L} + \underbrace{(c_V^e - c_A^e)^2}_{c_R} (1-y)^2 \Big]$$

and after integrating over y (or d cosθ)

$$\sigma^{NC}(\nu_\mu e^- \to \nu_\mu e^-) = \frac{G_F^2 s}{3\pi} \left[\left(c_V^e\right)^2 + c_V^e c_A^e + \left(c_A^e\right)^2 \right]$$

$$\sigma^{NC}(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-) = \frac{G_F^2 s}{3\pi} \left[\left(c_V^e\right)^2 - c_V^e c_A^e + \left(c_A^e\right)^2 \right]$$

And now, we can derive the full $\nu_e e^-$ scattering amplitude!
Both the CC (W exchange) and NC (Z exchange) channels contribute:
add the amplitudes $M = M^{CC}(\nu_e e^- \to e^- \nu_e) + M^{NC}(\nu_e e^- \to \nu_e e^-)$

$$M(\nu_e e^- \to \nu_e e^-) = \frac{4G_F}{\sqrt{2}}\left(\bar{u}_e\, \gamma^\mu \tfrac{1}{2}(1-\gamma^5)u_e\right)\left(\bar{u}_\nu\, \gamma_\mu \tfrac{1}{2}(1-\gamma^5)u_\nu\right) + \quad \textbf{CC}$$

$$\frac{4G_F}{\sqrt{2}} 2\rho\left(\bar{u}_e\, \gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)u_e\right)\frac{1}{2}\left(\bar{u}_\nu\, \gamma_\mu \tfrac{1}{2}(1-\gamma^5)u_\nu\right) \quad \textbf{NC}$$

Adding the amplitudes ($\rho = 1$ and $G_{NC} = G_F$)

$$M(\nu_e e^- \to \nu_e e^-) = \frac{4G_F}{\sqrt{2}}\left(\bar{u}_e\, \gamma^\mu \tfrac{1}{2}(c_V^e + 1 - (c_A^e+1)\gamma^5)u_e\right)\left(\bar{u}_\nu\, \gamma_\mu \tfrac{1}{2}(1-\gamma^5)u_\nu\right)$$

(i.e. equivalent to replace $c_V^e \to c_V^e + 1$ and $c_A^e \to c_A^e + 1$ in the NC amplitude)

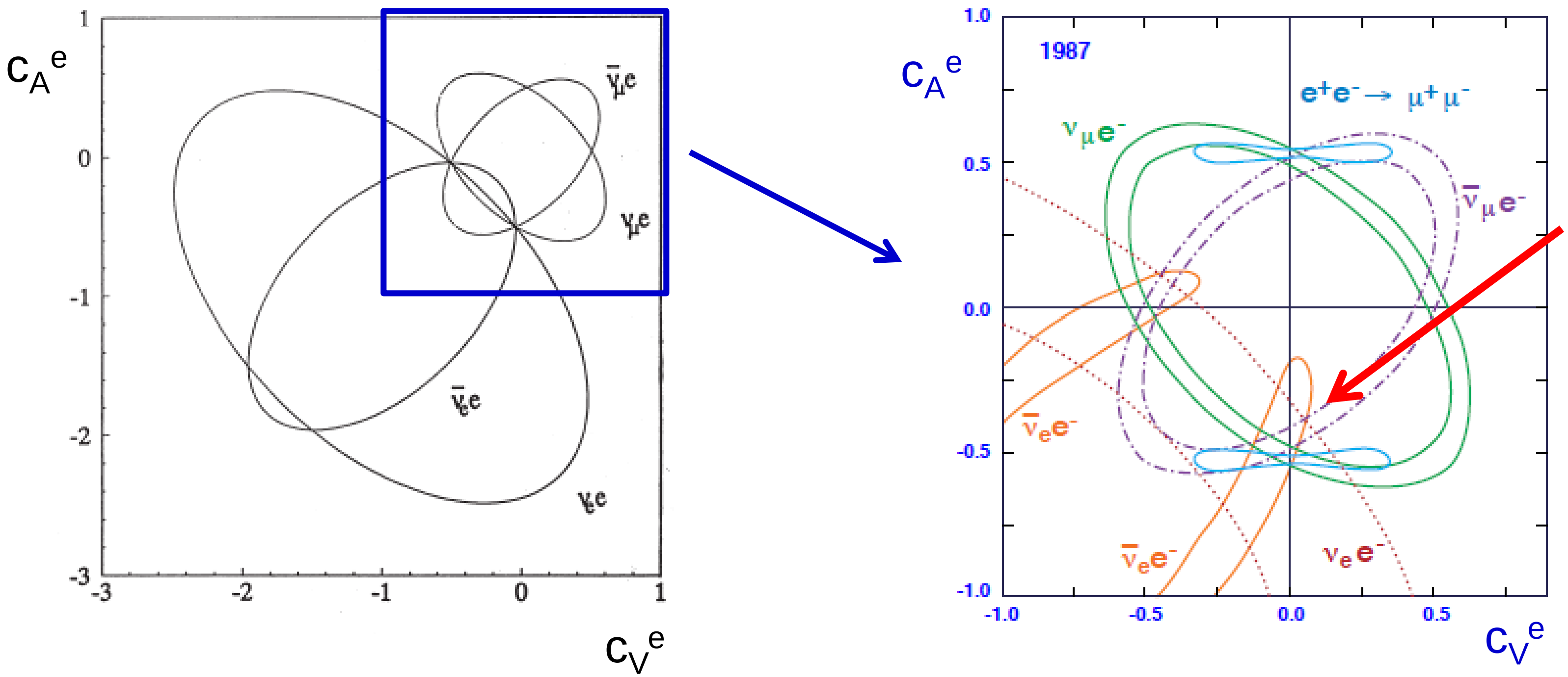
finally leads to

$$\frac{\mathrm{d}\sigma(\nu_e e^- \to \nu_e e^-)}{\mathrm{d}y} = \frac{G_F^2 s}{4\pi}\left[(c_V^e + c_A^e + 2)^2 + (c_V^e - c_A^e)^2(1-y)^2\right]$$

$$\sigma(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{4\pi}\left[(c_V^e + c_A^e + 2)^2 + \frac{1}{3}(c_V^e - c_A^e)^2\right]$$

equation of an ellipse in (c_V, c_A)

NC Parameters



only one point where all 5 "ellipses" overlap

In the Standard Model all c_V^i and c_A^i (i = ν, l, u or d quarks) expressed in terms of 1 parameter θ_W

$c_V^e = -1/2 + 2\sin^2\theta_W$ $\quad$ $c_V^\nu = +1/2$

$c_A^e = -1/2$ $\quad$ $c_A^\nu = +1/2$

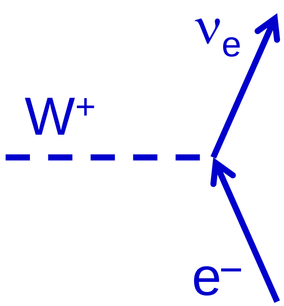
c_V^i and c_A^i are determined experimentally (including e^+e^- scattering at the Z^0 peak)

$$c_V^l = -0.03772 \pm 0.00041$$

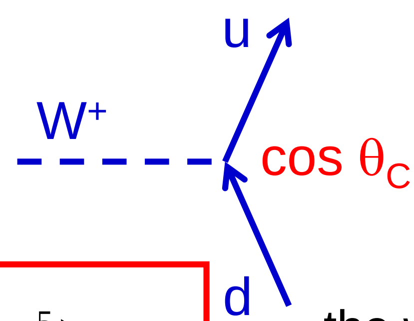
$$c_A^l = -0.50117 \pm 0.00027$$

$$c_V^\nu = c_A^\nu = +0.50085 \pm 0.00075$$

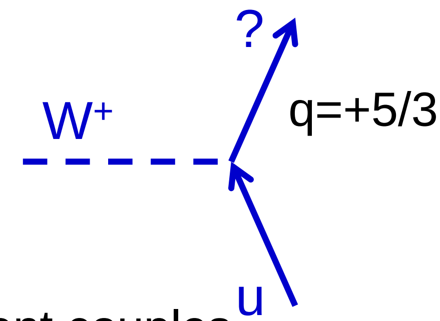
CC ν–q Scattering



charge rising

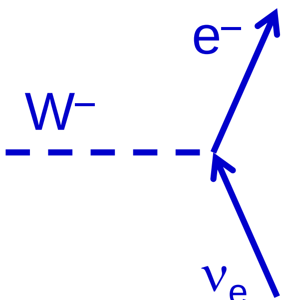


but not

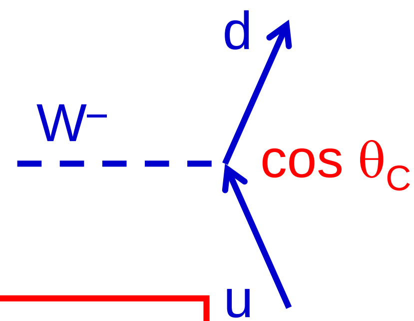


$$J_q^{\mu} = \bar{u}_u \gamma^{\mu} \frac{1}{2}(1-\gamma^5) V_{ud} u_d$$

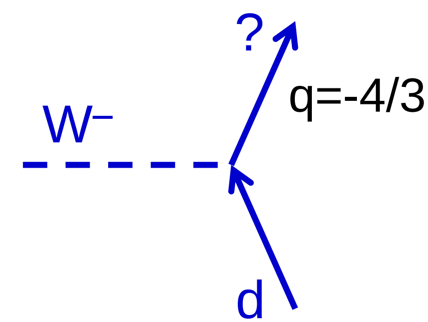
the weak CC current couples
d' = d cosθ_C + s sinθ_C to u quarks
θ_C is the Cabbibo angle, $\theta_C \sim 13.1^0$



charge lowering



but not



$$\left(J_q^{\mu}\right)^{\dagger} = \bar{u}_d V_{ud}^{*} \gamma^{\mu} \frac{1}{2}(1-\gamma^5) u_u$$

W⁺ couples to d and $\bar{u}$ quarks, but not to u nor to $\bar{d}$ quarks

W⁻ couples to u and $\bar{d}$ quarks, but not to d nor to $\bar{u}$ quarks (very selective!)

V – A structure: left handed (~negative helicity) u and d quarks
right handed (~positive helicity) u and d anti-quarks

CC ν–q Cross Sections

Follow the same arguments as for ν e scattering,
assume same coupling for quarks and for leptons

$$M = \frac{4G_F}{\sqrt{2}}\left(\bar{u}_{(\mu)}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)u_{(\nu)}\right)\left(\bar{u}_{(u)}\gamma_{\mu}\frac{1}{2}(1-\gamma^5)V_{ud}u_{(d)}\right)$$

$J_{\mu}^{\dagger}$ – charge lowering weak current

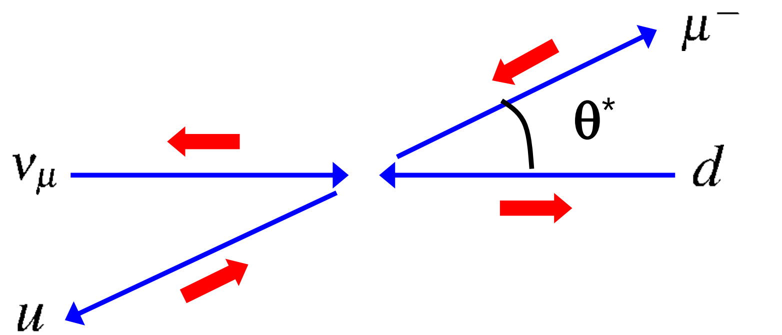
J^{μ} – charge raising weak current

$$\frac{d\sigma(\nu_\mu d \to \mu^- u)}{d\Omega} = \frac{G_F^2 V_{ud}^2}{4\pi^2}\hat{s}$$

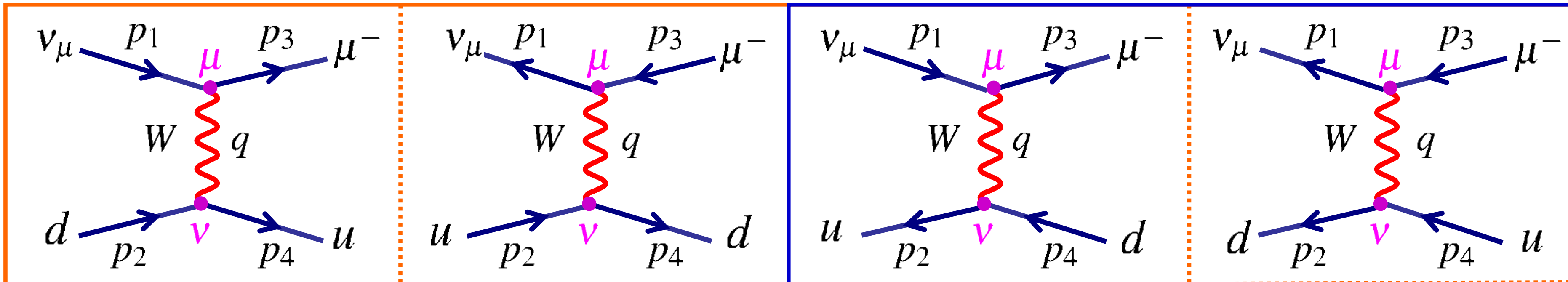
$$\frac{d\sigma(\bar{\nu}_\mu u \to \mu^+ d)}{d\Omega} = \frac{G_F^2 V_{ud}^2}{16\pi^2}\hat{s}(1+\cos\vartheta)^2$$

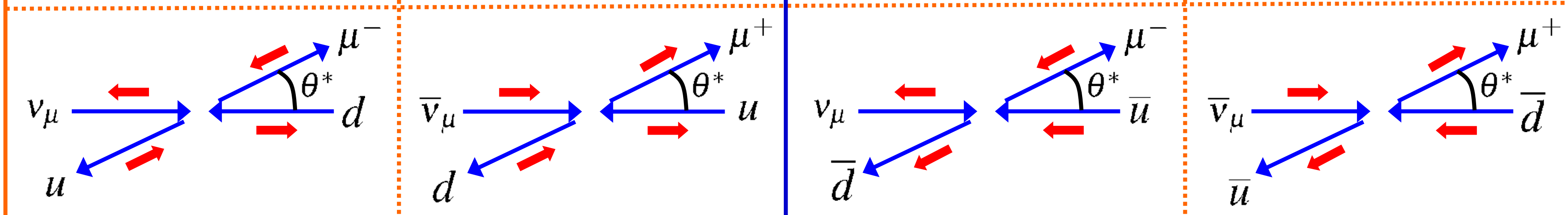
$$\frac{d\sigma(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u})}{d\Omega} = \frac{G_F^2 V_{ud}^2}{4\pi^2}\hat{s}$$

$$\frac{d\sigma(\nu_\mu \bar{u} \to \mu^- \bar{d})}{d\Omega} = \frac{G_F^2 V_{ud}^2}{16\pi^2}\hat{s}(1+\cos\vartheta)^2$$



Summary ν–q Scattering



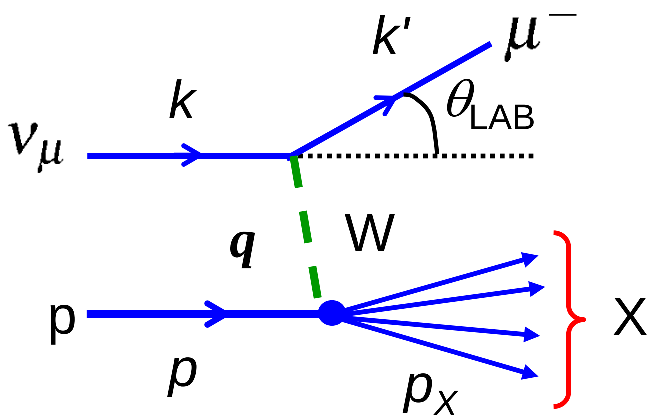


$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\overline{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\overline{\nu}\overline{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\overline{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu\overline{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\overline{\nu}\overline{q}} = \frac{G_F^2\hat{s}}{\pi}$
isotropic	no backward sc.	no backward sc.	isotropic

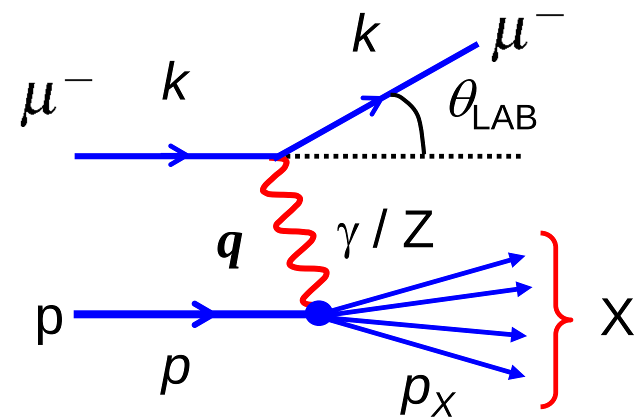
CC ν–N Scattering

To study ν-q scattering study ν-N interactions:

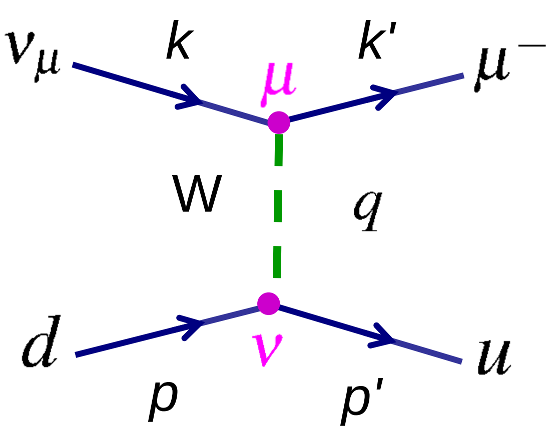
- quarks embedded in nucleons, similar formalism to μ-DIS
- additional information on parton distribution functions (quark's flavor!)
- ν beams mainly ν_μ from π decays produced at accelerators by a high intensity p beam



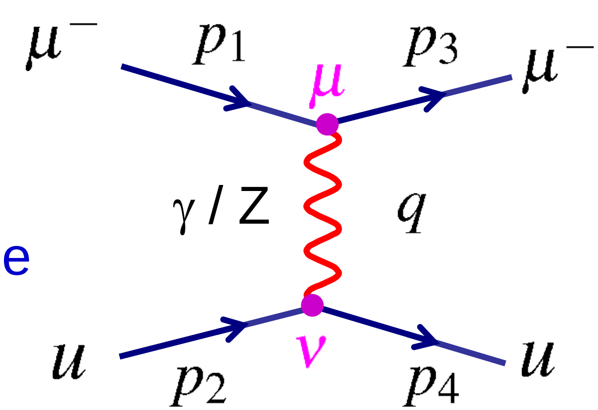
cfr. E.M.
1 γ exchange



Quarks embedded in nucleons, describe ν-N in terms of ν-q scattering.

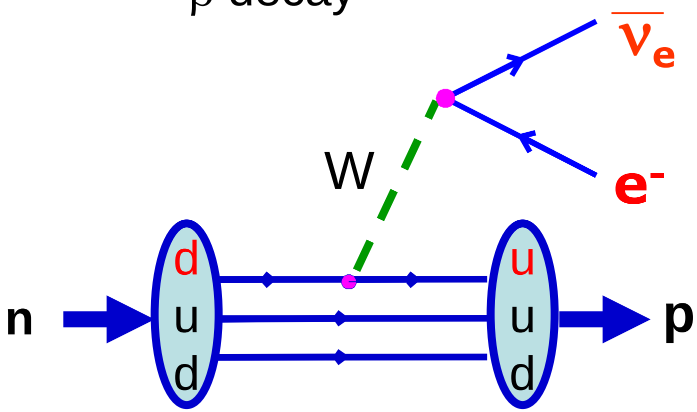


large Q^2
quarks are ~free



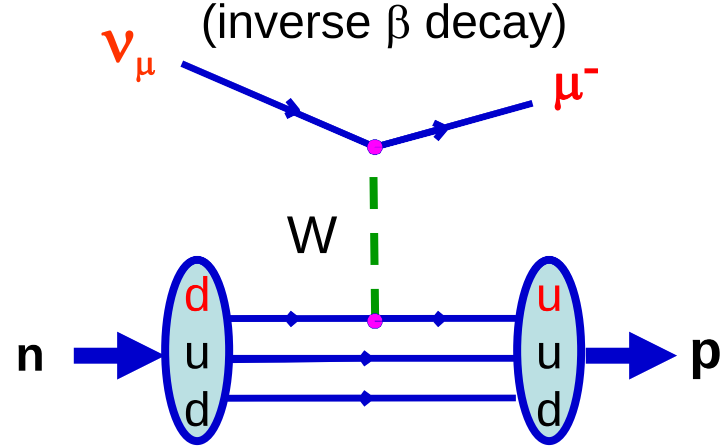
$$M = \frac{4G_F}{\sqrt{2}}\left(\bar{u}_{(\mu)}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^5)u_{(\nu)}\right)\left(\bar{u}_{(u)}\gamma_{\mu}\,\tfrac{1}{2}(1-\gamma^5)V_{ud}\,u_{(d)}\right)$$

β decay

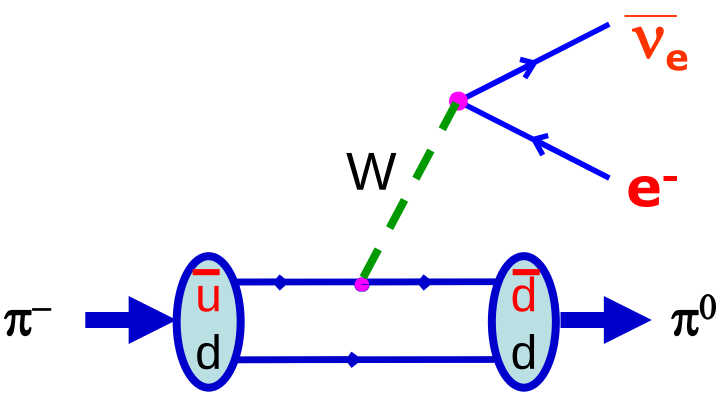


$$d \rightarrow u e^- \bar{\nu}_e$$

ν – N scattering
(inverse β decay)



$$\nu_\mu d \rightarrow \mu^- u$$



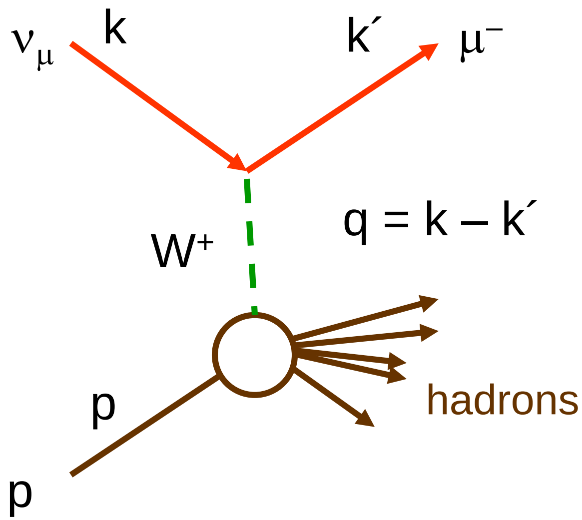
$$\bar{u} \rightarrow \bar{d} e^- \bar{\nu}_e$$

$$\Gamma\left(\pi^- \rightarrow \pi^0 e^- \bar{\nu}_e\right) = \frac{G_F^2}{30\pi^3}(\Delta m)^5$$

$$BR \sim 1\times 10^{-8}$$

Different from $\pi^- \rightarrow \mu^- \nu_\mu$ decay, in this case the d quark is a spectator and we can treat the $\bar{u}$ quark as free

$\nu - N$ Scattering



The energy of the incoming ν_μ beam not well determined

Measure scattered μ energy E_μ and angle θ and (possibly) all the final state hadrons (calorimeter)

infer ν_μ energy from scattered lepton plus final state hadron system (X)

$$E_\nu = E_\mu + E_{had}$$

kinematical variables

$$Q^2 = -q^2 = 4\left(E_{had} + E_\mu\right)E_\mu \sin^2\frac{\vartheta}{2}$$

$$x = \frac{\left(E_{had} + E_\mu\right)E_\mu \sin^2\frac{\vartheta}{2}}{2M_N E_{had}}$$

$$\nu = E_\nu - E_\mu = E_{had}$$

$$y = \frac{E_{had}}{E_{had} + E_\mu} \cong \frac{1}{2}\left(1 - \cos\vartheta^*\right)$$

$$W^2 = M_N^2 + 2M_N E_{had} + Q^2$$

The Experiments

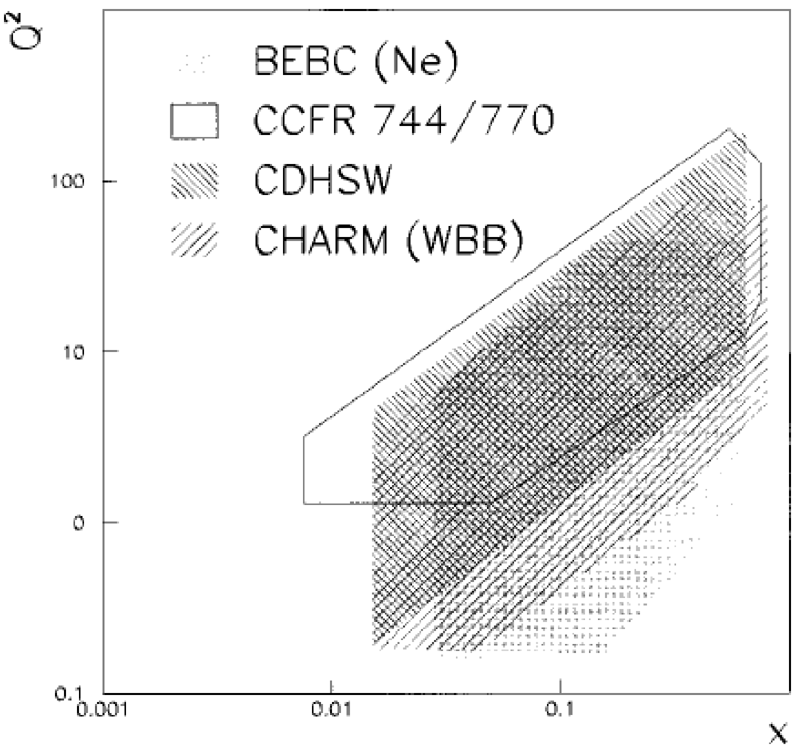
for a given E_ν,
kinematical region bounded by

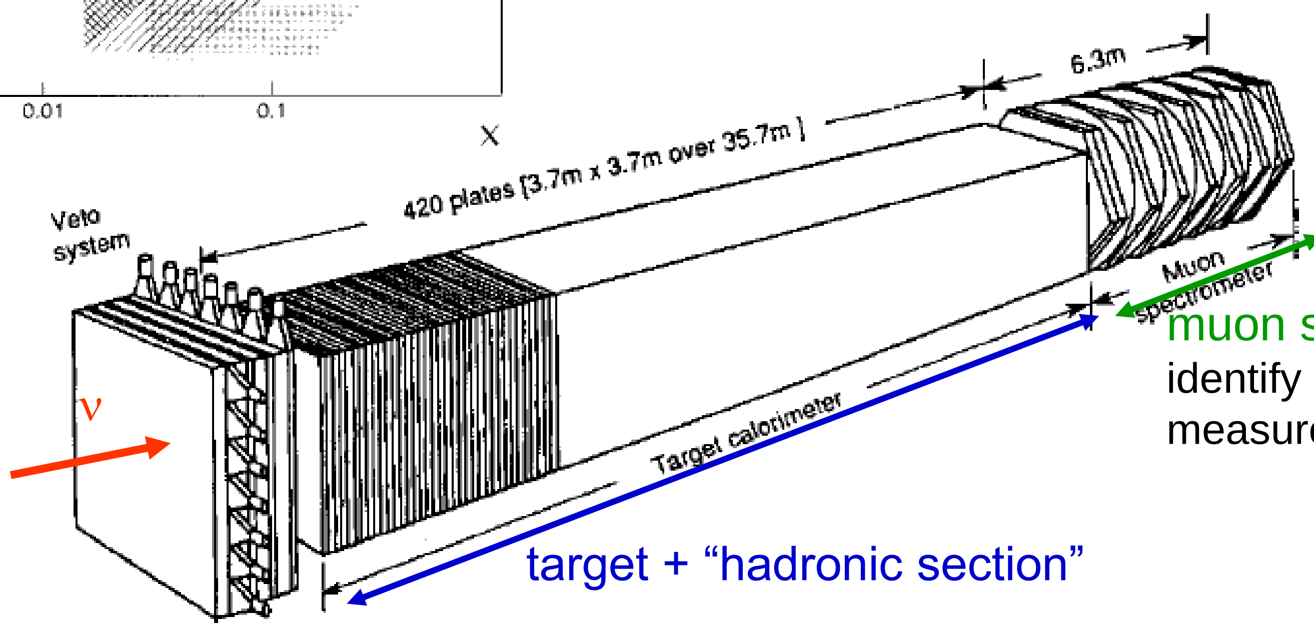
$$\frac{m_\mu^4}{8xME_\nu^2} \leqslant \nu \leqslant \frac{E_\nu}{(1+2Mx/E_\nu)} \rightarrow E_\nu,$$

$$\frac{m_\mu^4}{8xME_\nu^3} \leqslant y \leqslant \frac{1}{(1+2Mx/E_\nu)} \rightarrow 1,$$

$$\frac{m_\mu^4}{4E_\nu^2} \leqslant Q^2 \leqslant \frac{2ME_\nu x}{(1+2Mx/E_\nu)} \rightarrow 2ME_\nu x,$$

$$\frac{m_\mu^2}{2ME_\nu} \leqslant x \leqslant 1.$$





CHARM experiment at CERN

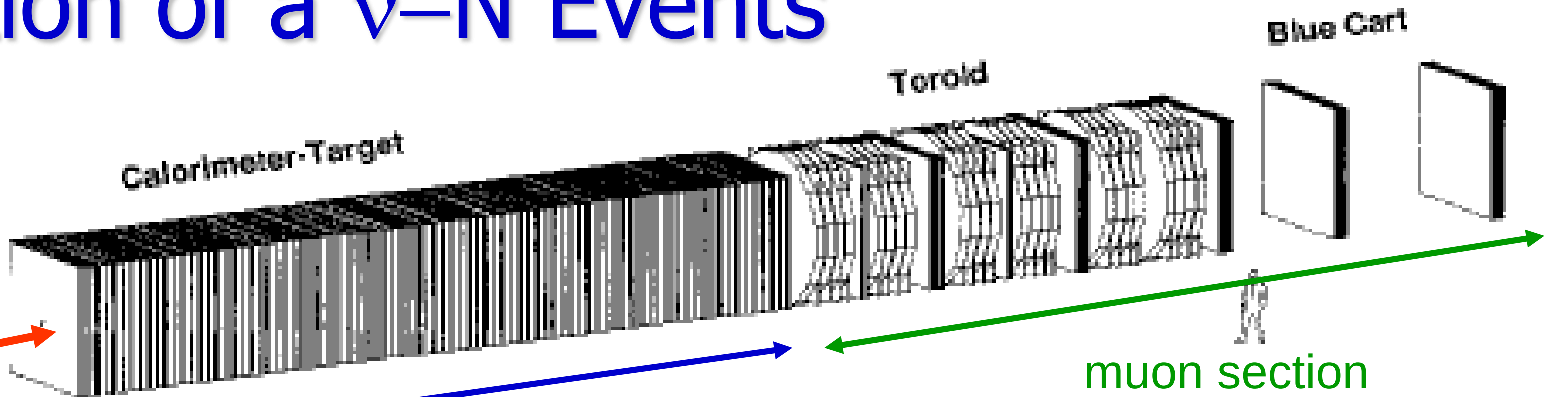
muon section
identify μ and measure E_μ

target + “hadronic section”

Detection of a ν–N Events

CCFR experiment at FNAL

ν

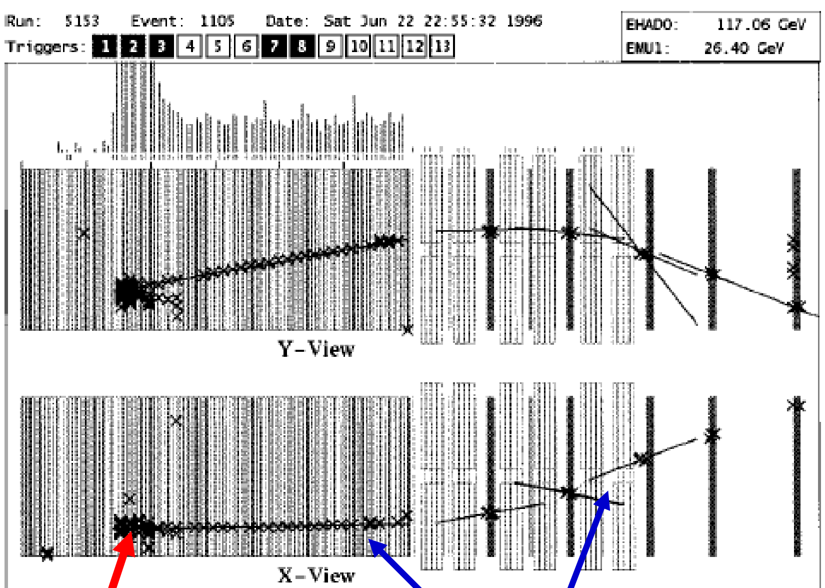


target + “hadronic section”

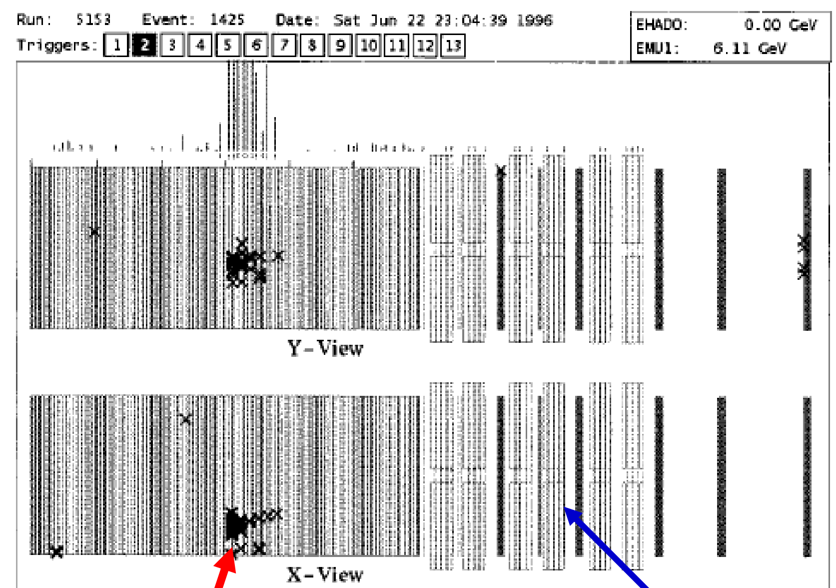
muon section

identify μ and measure E_μ

charged current event



neutral current event



hadronic shower

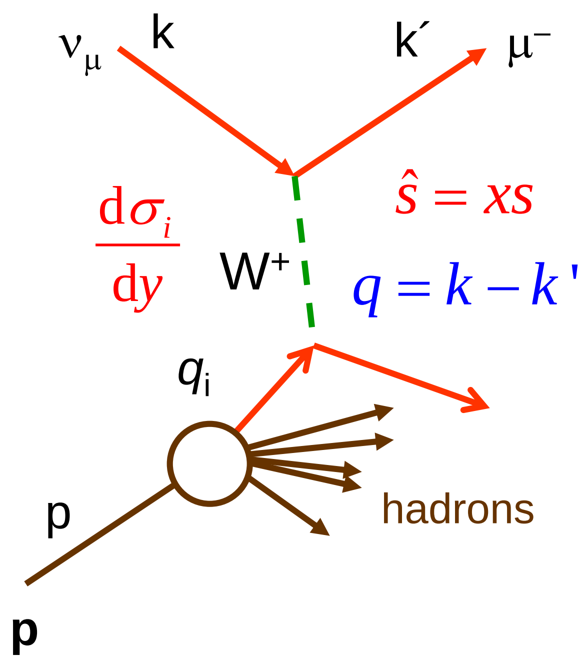
muon track

hadronic shower

no muon track

ν–N cross section

$$\frac{d\sigma(\nu_\mu N \to \mu^- X)}{dx\,dy} = \sum_i f_i(x)\frac{d\sigma_i(\nu_\mu q_i \to \mu^- q_i')}{dy}\Big|_{\hat{s}=xs}$$



Let consider an isoscalar target (same # of p and n) and let Q(x) and $\bar{Q}$(x) represent the probability to find a quark or an anti-quark in the nucleon with momentum *x*

$$Q(x) = d^p(x) + d^n(x) = d(x) + u(x)$$

$$\bar{Q}(x) = \bar{u}^p(x) + \bar{u}^n(x) = \bar{u}(x) + \bar{d}(x)$$

then

$$\frac{d\sigma(\nu_\mu N \to \mu^- X)}{dx\,dy} = \frac{G_F^2}{\pi} xs \frac{1}{2}\left[Q(x) + (1-y)^2 \bar{Q}(x)\right]$$

and

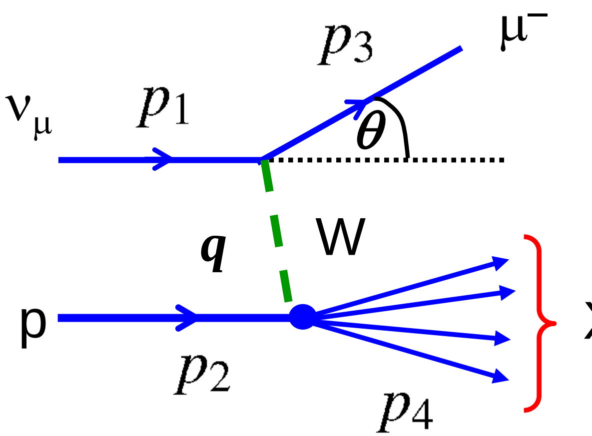
$$\frac{d\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dx\,dy} = \frac{G_F^2}{\pi} xs \frac{1}{2}\left[\bar{Q}(x) + (1-y)^2 Q(x)\right]$$

(origin of 1/2: we assumed an isoscalar target N = (p+n) / 2)

cfr. QED
$$\frac{d\sigma(e^\pm N \to e^\pm X)}{dx\,dy} = \frac{4\pi\alpha^2}{q^4} xs \frac{1}{2}\left[1 + (1-y)^2\right]\frac{5}{18}\left[Q(x) + \bar{Q}(x)\right]$$

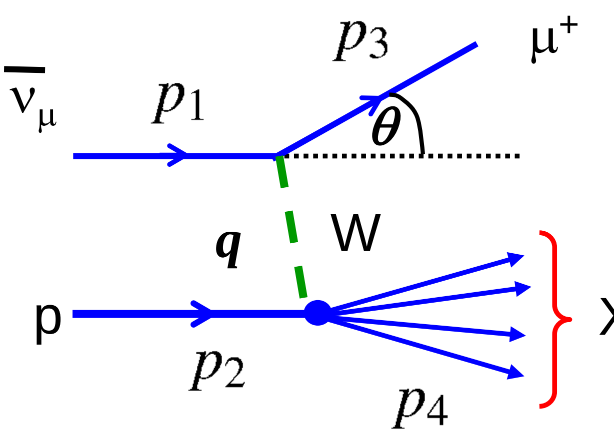
Where are the $\bar{q}$'s?

Study the angular distribution of the outgoing charged lepton in ν – DIS scattering



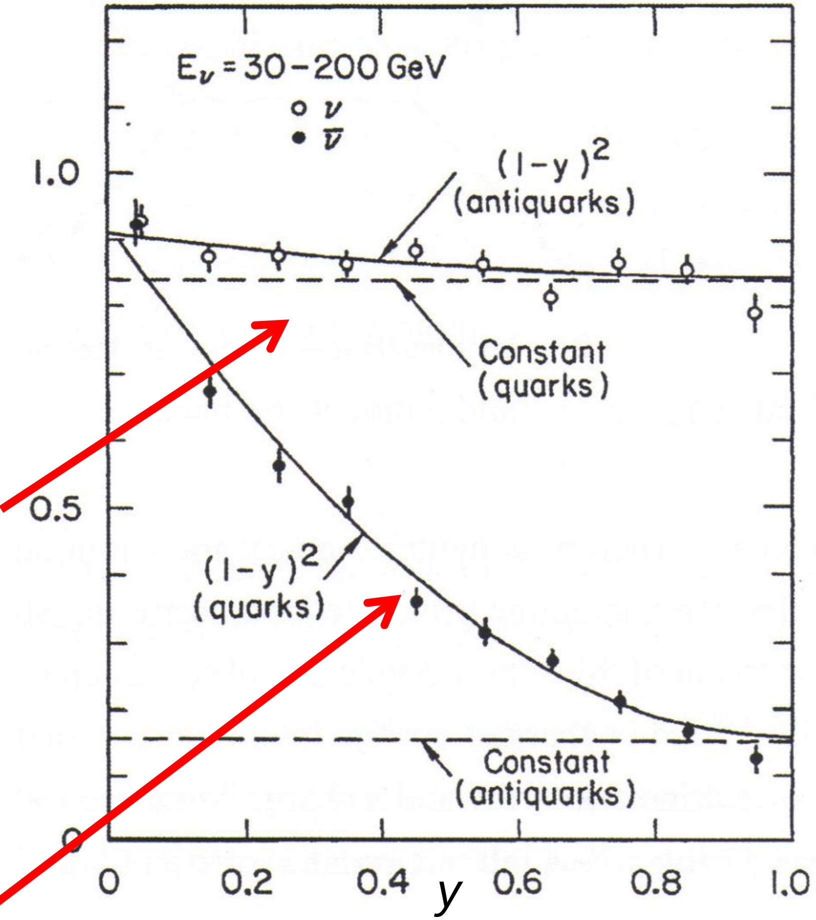
if only quarks

no *y* dependence
(i.e. isotropic)



if only quarks

$\propto (1 - y)^2$
(i.e. no backward scattering)



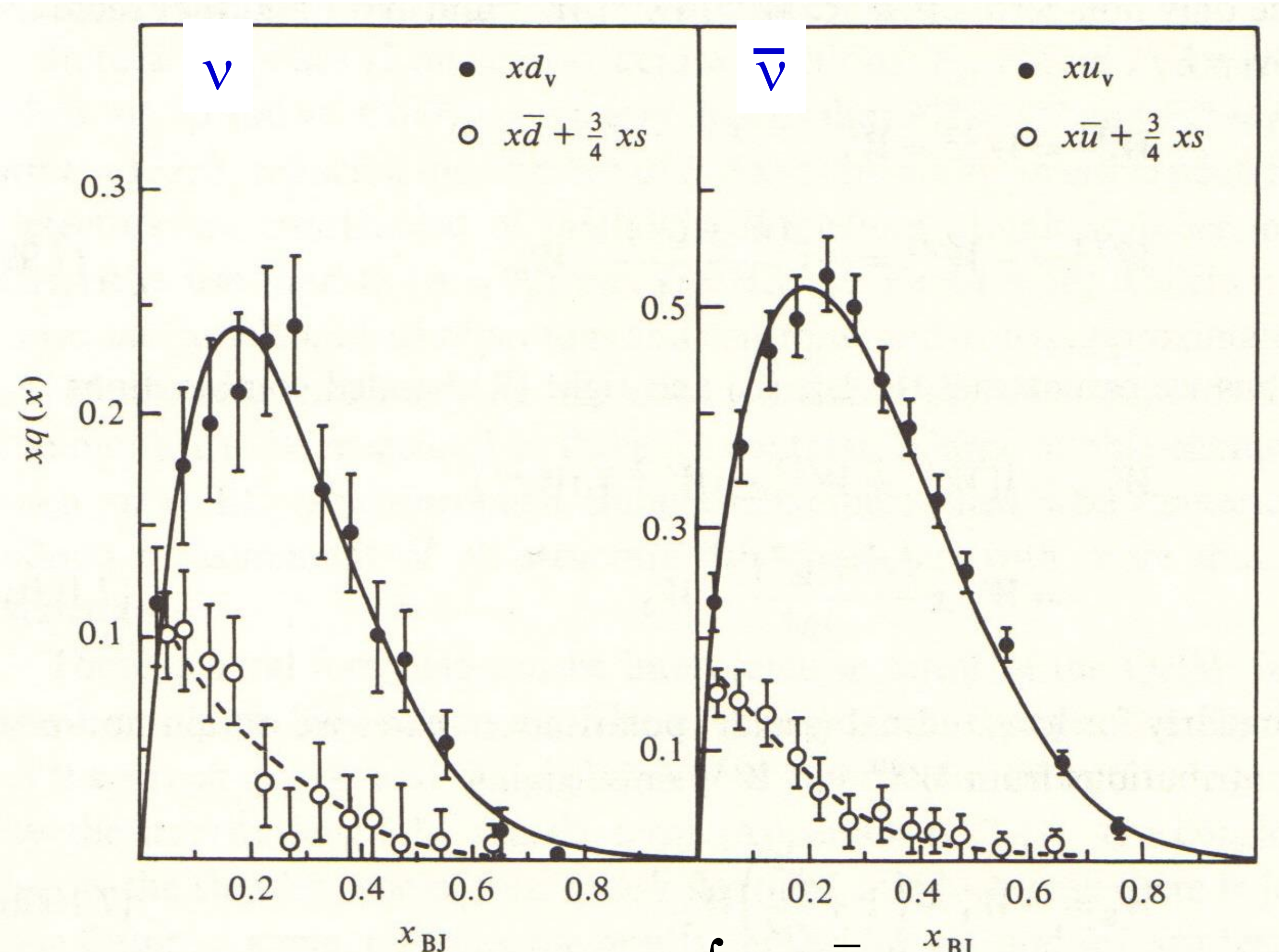
$$\frac{d\sigma(\nu q)}{dy} = \frac{G_F^2}{\pi} s \qquad \frac{d\sigma(\bar{\nu} q)}{dy} = \frac{G_F^2}{\pi} (1-y)^2 s$$

$$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{1}{3}$$

comparison of ν and $\bar{\nu}$ angular distribution allows one to separate of q and $\bar{q}$:
one finds a small angular dependence in νN and a small flat component in $\bar{\nu}$N
⇒ consistent with ~ 5% antiquarks in nucleon

Quark Distributions

Valence and Sea quark distributions
extracted from ν (anti-ν) D interactions (Q^2 ~ 5 GeV^2)



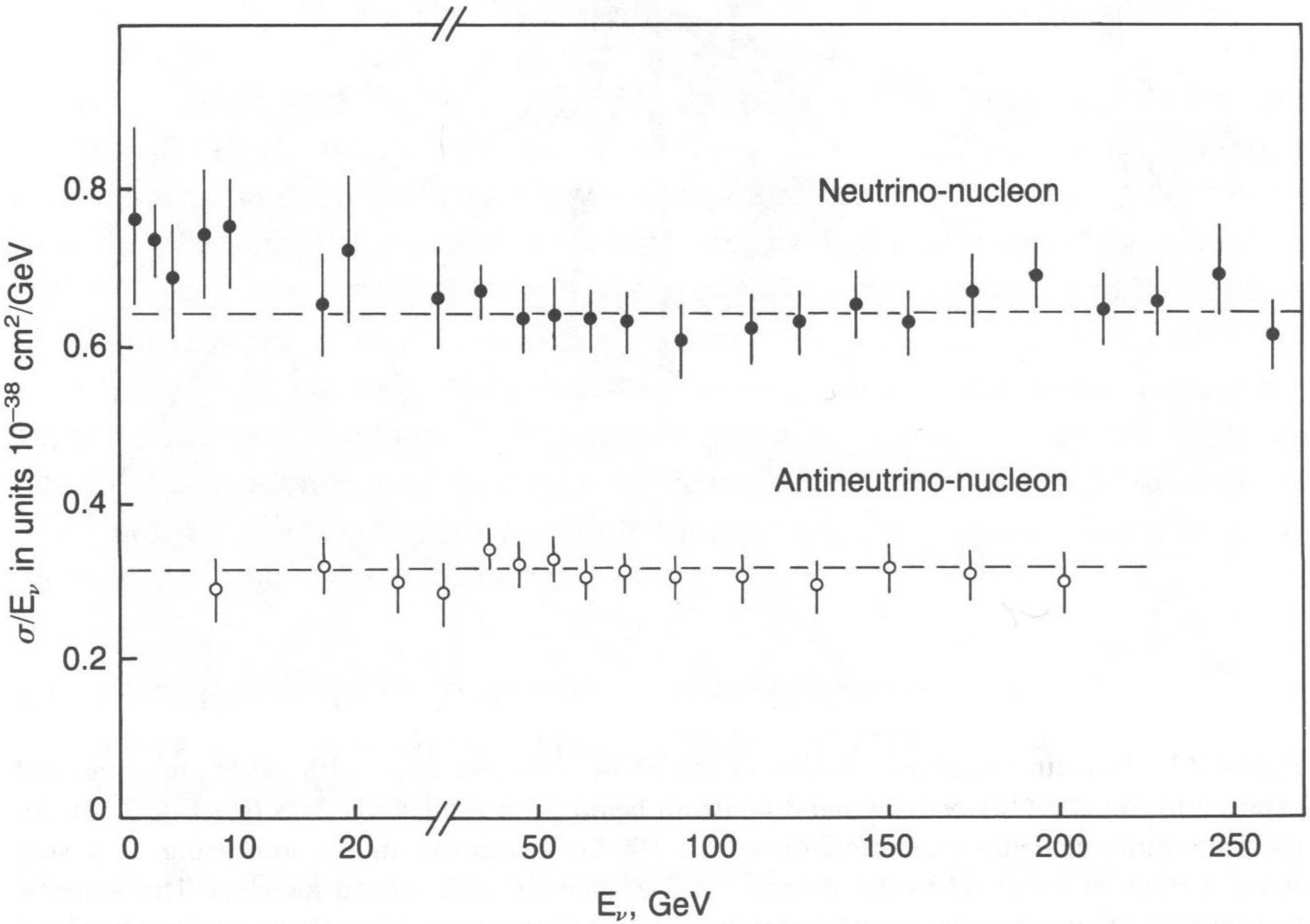
quark and anti-quark distributions at Q^2 ~ 10 GeV^2

~ 5% $\bar{Q}$ component in proton

In general, if $$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = R \Rightarrow \frac{\int dx\ x\bar{Q}(x)}{\int dx\ xQ(x)} = \frac{3R-1}{3-R}$$

ν – N Total Cross Section Data

$$\sigma^{\nu N} = \frac{G_F^2}{2\pi} s \left[Q + \frac{1}{3}\bar{Q} \right] \qquad \sigma^{\bar{\nu} N} = \frac{G_F^2}{2\pi} s \left[\frac{1}{3} Q + \bar{Q} \right] \qquad Q = \int_0^1 \mathrm{d}x \; x\left[u(x) + d(x) + s(x)\right]$$



not exactly a factor of 3 because of anti-quarks in the nucleon

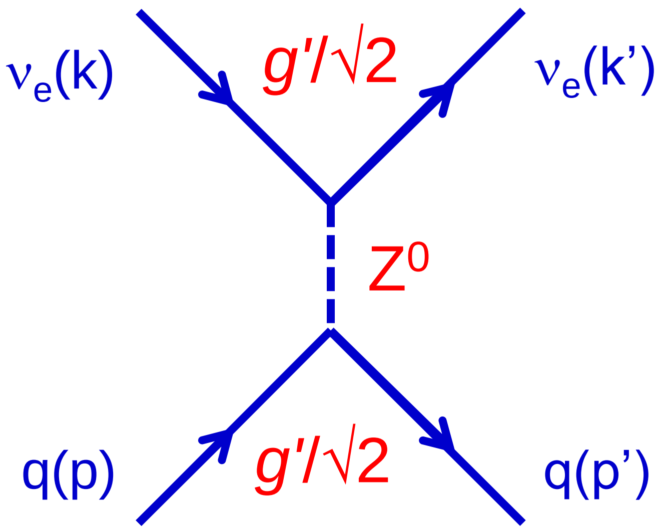
$$\frac{\sigma(\nu N)}{\sigma(\bar{\nu} N)} \approx 2$$

NC ν – q Scattering

Because $\nu_e e$ cross sections are very small, extensive studies of NC interactions carried out using isoscalar (#p = #n) nuclear targets, like Fe (almost isoscalar)

$$R_\nu = \frac{\sigma^{NC}(\nu)}{\sigma^{CC}(\nu)} = \frac{\sigma^{NC}(\nu_\mu N \to \nu_\mu X)}{\sigma^{CC}(\nu_\mu N \to \mu^- X)} = 0.31 \pm 0.01$$

$$R_{\bar{\nu}} = \frac{\sigma^{NC}(\bar{\nu})}{\sigma^{CC}(\bar{\nu})} = \frac{\sigma^{NC}(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma^{CC}(\bar{\nu}_\mu N \to \mu^+ X)} = 0.38 \pm 0.02$$



Explained in terms of νq – νq and $\bar{\nu}$q – $\bar{\nu}$q scattering

$$\frac{d\sigma^{NC}(\nu q \to \nu q)}{dy} = \frac{G_F^2 xs}{\pi}\left[\frac{1}{4}(c_V^q + c_A^q)^2 + \frac{1}{4}(c_V^q - c_A^q)^2(1-y)^2\right]$$

$$c_L = \frac{1}{2}(c_V^q + c_A^q) \qquad c_R = \frac{1}{2}(c_V^q - c_A^q)$$

$$\frac{d\sigma^{NC}(\nu N \to \nu X)}{dxdy} = \frac{G_F^2 xs}{2\pi}\left[c_L^2[Q(x) + (1-y)^2\bar{Q}(x)] + c_R^2[\bar{Q}(x) + (1-y)^2 Q(x)]\right]$$

experimentally $\quad c_L^2 = 0.300 \pm 0.015 \qquad c_R^2 = 0.024 \pm 0.008$

The ν – N Cross Section

The calculation proceeds in a similar way as eμ → eμ, ep → ep, ep → eX cross sections

lepton current $j^{\mu} = \bar{u}_{(\mu)}\gamma^{\mu}\frac{1}{2}(1-\gamma_5)u_{(\nu)}$ (both V and A currents possible)

hadron current $J^{\mu}_{had} = \langle X | J^{\mu}_{W} | p, s \rangle$

invariant amplitude $M = \sqrt{2}G_F\, j_{\mu}J^{\mu} \rightarrow \sqrt{2}G_F \frac{1}{1+Q^2/M_W^2} j_{\mu}J^{\mu}_{had}$

W propagator $\frac{-i\left(g_{\mu\nu} - q_{\mu}q_{\nu}/M_W^2\right)}{q^2 - M_W^2}$

leptonic tensor

$$L_{\alpha\beta} = \left[\bar{u}(k)\gamma_{\alpha}(1-\gamma^5)u(k')\right]\left[\bar{u}(k')\gamma_{\beta}(1-\gamma^5)u(k)\right]$$
$$= 8\left[k'_{\alpha}k_{\beta} + k'_{\beta}k_{\alpha} - (k\cdot k' - m_{\mu}^2)g_{\alpha\beta} \mp i\varepsilon_{\alpha\beta\gamma\delta}k^{\gamma}k'^{\delta}\right]$$

hadronic tensor
(most general form)

$$W^{\alpha\beta} = -g^{\alpha\beta}W_1 + \frac{p^{\alpha}p^{\beta}}{M^2}W_2 - \frac{i\varepsilon^{\alpha\beta\gamma\delta}p_{\gamma}q_{\delta}}{2M^2}W_3$$
$$+ \frac{q^{\alpha}q^{\beta}}{M^2}W_4 + \frac{p^{\alpha}q^{\beta} + p^{\beta}q^{\alpha}}{M^2}W_5 + i\frac{p^{\alpha}q^{\beta} - p^{\beta}q^{\alpha}}{2M^2}W_6$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the fully antisymmetric tensor

Contract the tensor $L_{\mu\nu}W^{\mu\nu}$

the terms W_4, W_5, and W_6 are proportional to lepton masses $\rightarrow 0$

the antisymmetric term W_3 stays, and violates parity

$$L_{\mu\nu}W^{\mu\nu} = 4W_1(k\cdot k') + \frac{2W_2}{M^2}\left[2(p\cdot k)(p\cdot k') - M^2(k\cdot k')\right] - \frac{2W_3}{M^2}\left[(p\cdot k)(q\cdot k') - (q\cdot k')(k\cdot p)\right]$$

The cross section in the lab frame becomes (ν experiments only fixed target so far …)

$$\left(\frac{d^2\sigma}{dxdy}\right)\begin{pmatrix}\nu N\\ \bar{\nu} N\end{pmatrix} = \frac{G_F^2 s}{2\pi\left(1+Q^2/M_W^2\right)^2}\left[\nu W_2\left(1-y-\frac{Mxy}{2E_\nu}\right)+\frac{y^2}{2}2xMW_1 \pm y\left(1-\frac{y}{2}\right)x\nu W_3\right]$$

For $\bar{\nu}$ N we replace $(1-\gamma_5)$ by $(1+\gamma_5)$ in the lepton tensor and this changes the sign of W_3.
Recall that E_ν is not directly measurable, need the total energy E_{had} of the hadronic final system

$$\left(\frac{d^2\sigma}{dQ^2 d\nu}\right)\begin{pmatrix}\nu N\\ \bar{\nu} N\end{pmatrix} = \frac{G_F^2}{2\pi M}\frac{E_\mu}{E_\nu}\left[W_2\cos^2\frac{\vartheta}{2}+2W_1\sin^2\frac{\vartheta}{2}\pm\left(\frac{E_\nu+E_\mu}{M}\right)W_3\sin^2\frac{\vartheta}{2}\right]$$

The structure functions W_1, W_2, W_3 are different functions from those encountered in e – p scattering, and also differ from ν N to $\bar{\nu}$ N scattering.
No assumption on the underlying structure of the hadron is made so far.

DIS Region

The structure functions W are functions of $W(Q^2,\nu)$.
In DIS region the probe sees the constituents inside the hadron (Bjorken scaling limit)

→ scattering off point-like partons

→ W depends on only 1 variable $x = Q^2/2M\nu$

(compare to eμ elastic scattering, QPM, …)

$$2MW_1(Q^2,\nu) = \frac{Q^2}{2M\nu}\delta\left(1-\frac{Q^2}{2M\nu}\right)$$

$$\nu W_2(Q^2,\nu) = \delta\left(1-\frac{Q^2}{2M\nu}\right)$$

$$\nu W_3(Q^2,\nu) = \delta\left(1-\frac{Q^2}{2M\nu}\right)$$

and the structure functions
In the Bjorken limit
take the familiar form

$$MW_1(Q^2,\nu) \to F_1^\nu(x)$$

$$\nu W_2(Q^2,\nu) \to F_2^\nu(x)$$

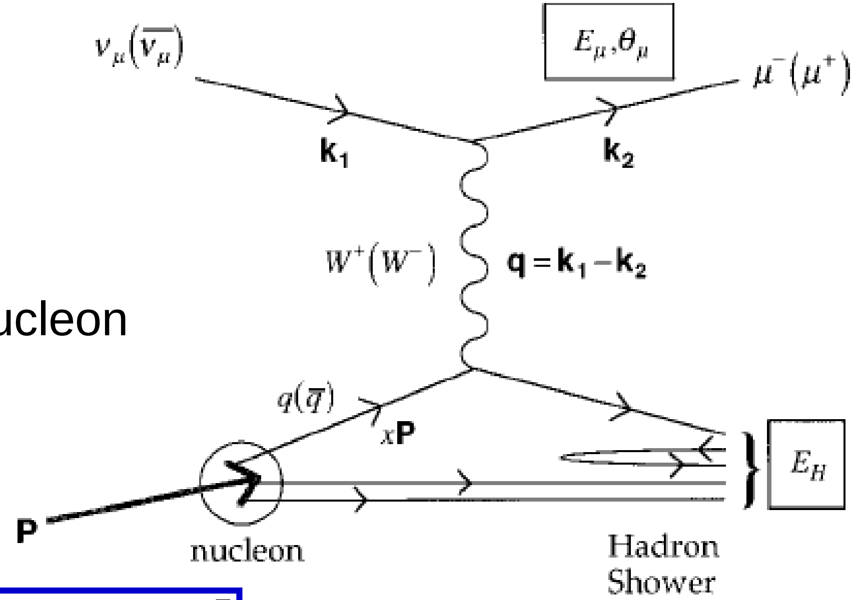
$$\nu W_3(Q^2,\nu) \to F_3^\nu(x)$$

In total there are 12 nucleon structure functions for neutrino scattering:
F_1, F_2, F_3 for each of the νp, νn, $\bar{\nu} p$, $\bar{\nu} n$ processes.

Charge symmetry implies $F_i(\nu p) = F_i(\bar{\nu} n)$ and $F_i(\nu n) = F_i(\bar{\nu} p)$, which reduces them to 6.

Experiments conducted usually using massive isoscalar targets (#p = #n) and measure cross sections per nucleon (p or n) reducing thus the number of structure functions to 3

QPM Interpretation



Interpretation similar to μN scattering

The exchanged $W^\pm$ strikes a quark in the nucleon and changes the flavor of the struck quark (probability given by CKM matrix)

QPM cross section (isoscalar target)

$$\frac{d^2\sigma}{dxdy}(\nu N) = \frac{G_F^2 M E_\nu}{\pi\left(1+Q^2/M_W^2\right)^2} 2x\left[\sum_i c_i^2 q_i(x) + \sum_j c_j^2 q_j(x)(1-y)^2\right]$$

c_i and c_j are the Cabibbo mixing angles (cos θ_C and sin θ_C, assuming 4 flavors only)

Assuming the Callan-Gross relation (spin ½ quarks) $F_2 = 2xF_1$ and an isoscalar target

$$F_2(x) = 2\sum_{i,j}\left[c_i^2 x q_i(x) + c_j^2 x \overline{q}_j(x)\right]$$

$$xF_3(x) = 2\sum_{i,j}\left[c_i^2 x q_i(x) - c_j^2 x \overline{q}_j(x)\right]$$

the striking difference between F_2 and F_3 is the sign for the anti-quark densities

and the cross section finally reads

$$\frac{d^2\sigma}{dxdy}\begin{pmatrix}\nu N \\ \overline{\nu} N\end{pmatrix} = \frac{G_F^2 s}{2\pi\left(1+Q^2/M_W^2\right)^2}\left[\frac{F_2 \pm xF_3}{2} + \frac{F_2 \mp xF_3}{2}(1-y)^2\right]$$

Quark Content of F_2 and F_3

νN Cabibbo favored ($\cos^2 \theta_C$) $\quad d \to u,\ s \to c,\ \bar{u} \to \bar{d},\ \bar{c} \to \bar{s}$

$\bar{\nu}$N Cabibbo unfavored ($\sin^2 \theta_C$) $\quad d \to c,\ s \to u,\ \bar{c} \to \bar{d},\ \bar{u} \to \bar{s}$

νN Cabibbo favored transitions $\quad u \to d,\ c \to s,\ \bar{d} \to \bar{u},\ \bar{s} \to \bar{c}$

$$
\begin{array}{lll}
\nu p: & F_2 = 2x\left[d + s + \bar{u} + \bar{c}\right] & F_3 = 2x\left[d + s - \bar{u} - \bar{c}\right] \\
\nu n: & F_2 = 2x\left[u + s + \bar{d} + \bar{c}\right] & F_3 = 2x\left[u + s - \bar{d} - \bar{c}\right] \\
\bar{\nu} p: & F_2 = 2x\left[u + c + \bar{d} + \bar{s}\right] & F_3 = 2x\left[u + c - \bar{d} - \bar{s}\right] \\
\bar{\nu} n: & F_2 = 2x\left[d + c + \bar{u} + \bar{s}\right] & F_3 = 2x\left[d + c - \bar{u} - \bar{s}\right]
\end{array}
$$

For an isocalar target (p+n)

$$
\begin{array}{lll}
\nu N: & F_2 = 2x\left[u + d + \bar{u} + \bar{d} + 2s + 2\bar{c}\right] & F_3 = 2x\left[u + d - \bar{u} - \bar{d} + 2s - 2\bar{c}\right] = 2x\left[u_V + d_v + 2(s - \bar{c})\right] \\
\bar{\nu} N: & F_2 = 2x\left[u + d + \bar{u} + \bar{d} + 2\bar{s} + 2c\right] & F_3 = 2x\left[u + d - \bar{u} - \bar{d} + 2c - 2\bar{s}\right] = 2x\left[u_V + d_v + 2(c - \bar{s})\right]
\end{array}
$$

since s = $\bar{s}$ and c = $\bar{c}$ (sea quarks)

$$F_2(\bar{\nu} N) = F_2(\nu N)$$

$$F_3(\bar{\nu} N) = F_3(\nu N) - 4x[s - c] \approx 2x[u_V + d_V]$$

(basically, F_3 measures the distribution of valence quarks)

Comments on F_2 and F_3

ν and $\bar{\nu}$ F_2 structure function on isoscalar targets ½(p+n) measures the SUM of quark and anti-quark PDFs in the nucleon

$$F_2^{\nu N} = F_2^{\bar{\nu} N} = 2x\left[u + d + \bar{u} + \bar{d} + 2\left(s + \bar{s}\right)\right]$$

Cfr. EM scattering

$$F_2^{ep} = x\left[\frac{4}{9}\left(u+\bar{u}\right) + \frac{1}{9}\left(d+\bar{d}\right) + \frac{1}{9}\left(s+\bar{s}\right)\right]$$

The average of xF_3 for ν and $\bar{\nu}$ scattering on isoscalar targets measure the VALENCE quark PDFs

$$\frac{F_3^{\nu N} + F_3^{\bar{\nu} N}}{2} = 2x\left[u + d - \bar{u} - \bar{d}\right] = 2x\left[u_V + d_v\right]$$

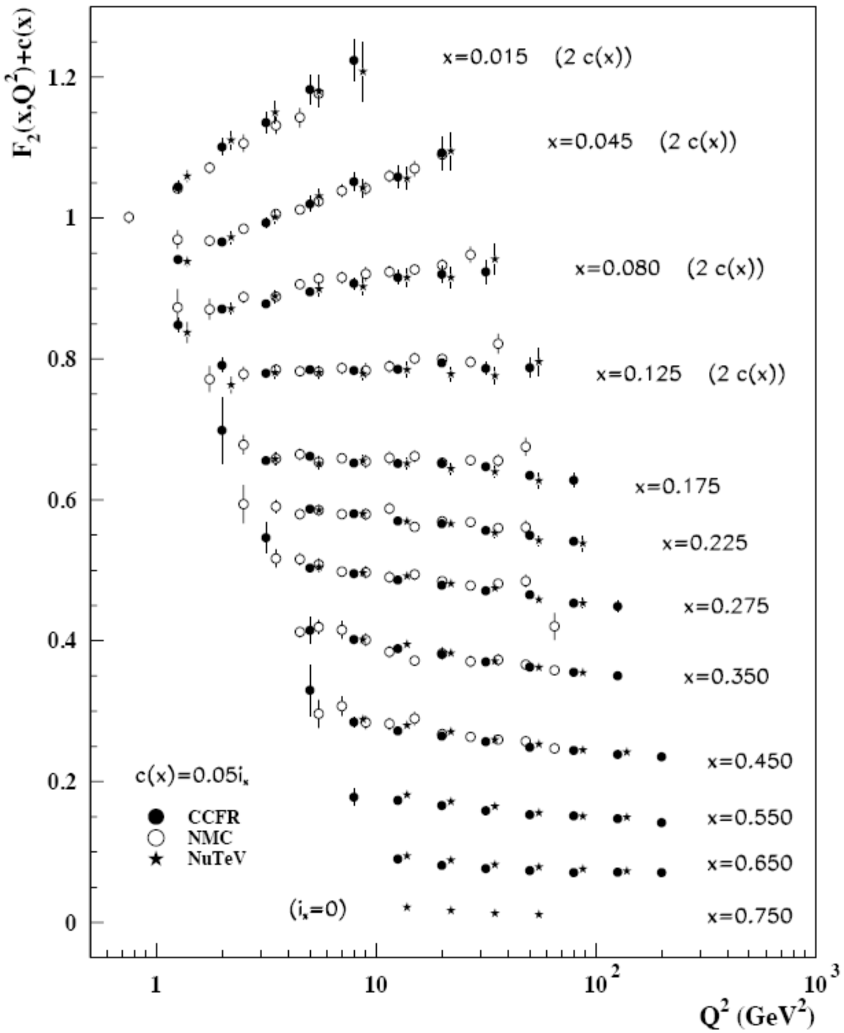
Comparison of F_2 for ν and e/μ lepton scattering confirms the fractional charge assignment to the quarks.

The difference $\Delta\left(xF_3\right) = xF_3\left(\nu N\right) - xF_3\left(\bar{\nu} N\right)$

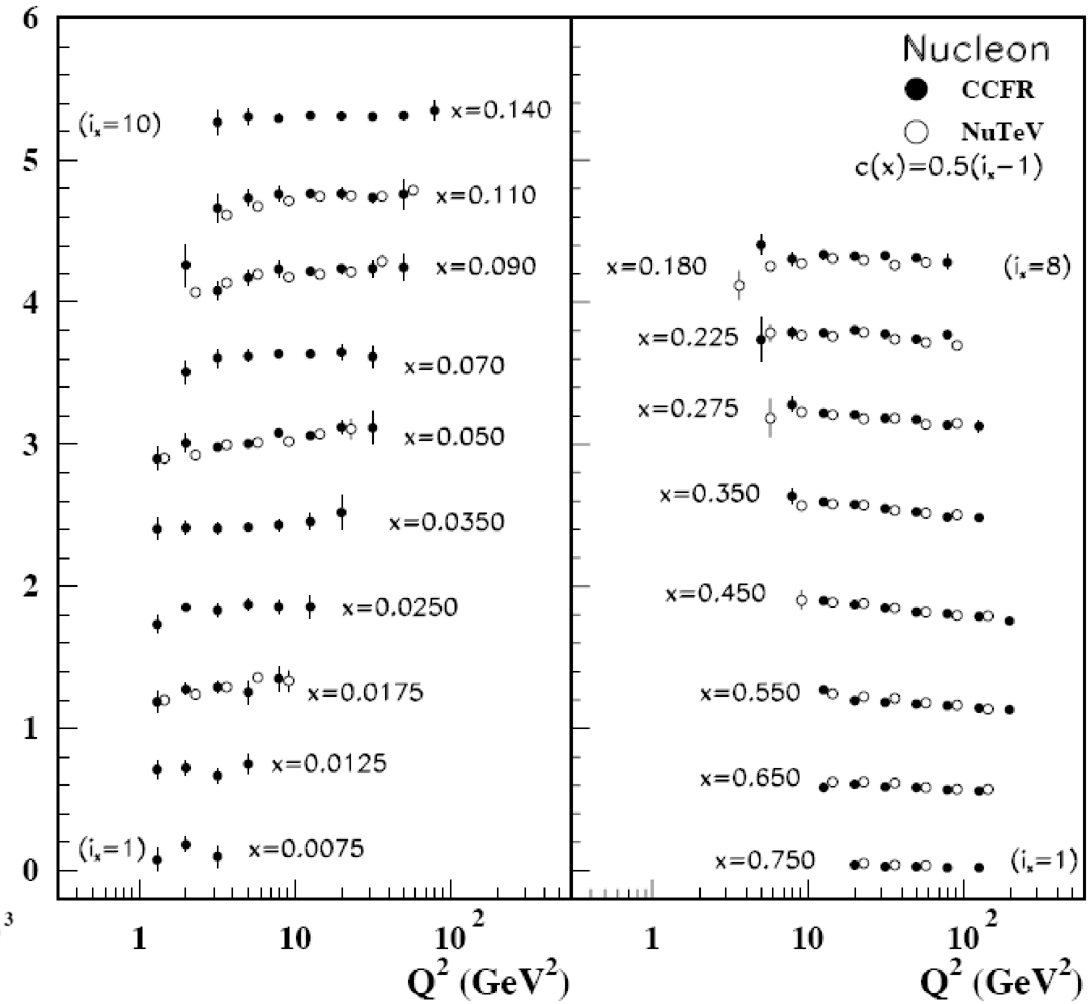
is sensitive to the strange and charm quark content of the nucleon

F_2 and F_3 Structure Functions in ν – Fe Scat.

F_2 ν Fe scattering



F_3 ν Fe scattering



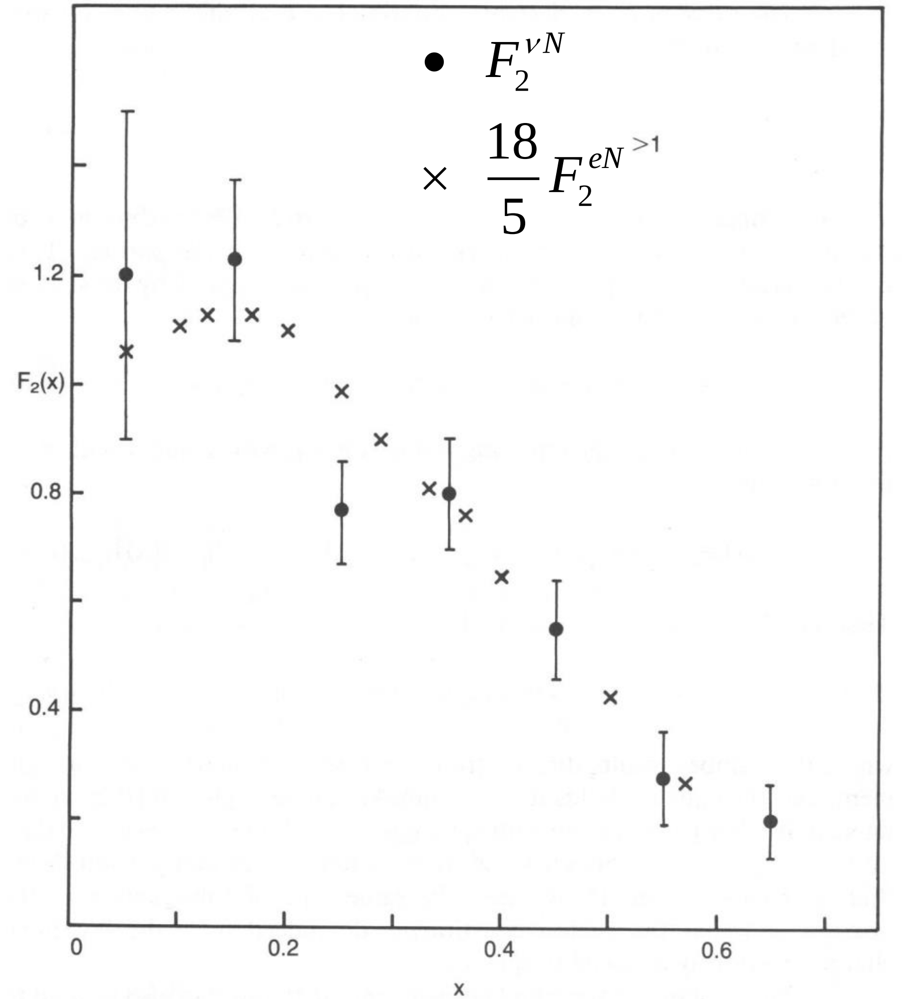
ν – e/μ Comparison

An early check of fractional charges comes from F_2 comparisons in eN and νN scattering off isoscalar targets

$$F_2^{e/\mu} = \frac{5}{18} x\left[u + d + \bar{u} + \bar{d} + \frac{2}{5}[s+\bar{s}] + \frac{8}{5}[c+\bar{c}]\right]$$

$$F_2^{\nu} = x\left[u + d + \bar{u} + \bar{d} + 2s + 2\bar{c}\right]$$

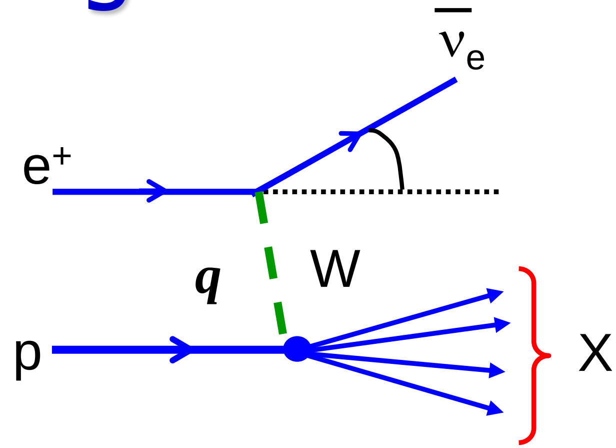
$$F_2^{e/\mu} = \frac{5}{18} F_2^{\nu} - x\frac{1}{6}\left(s+\bar{s}\right)$$



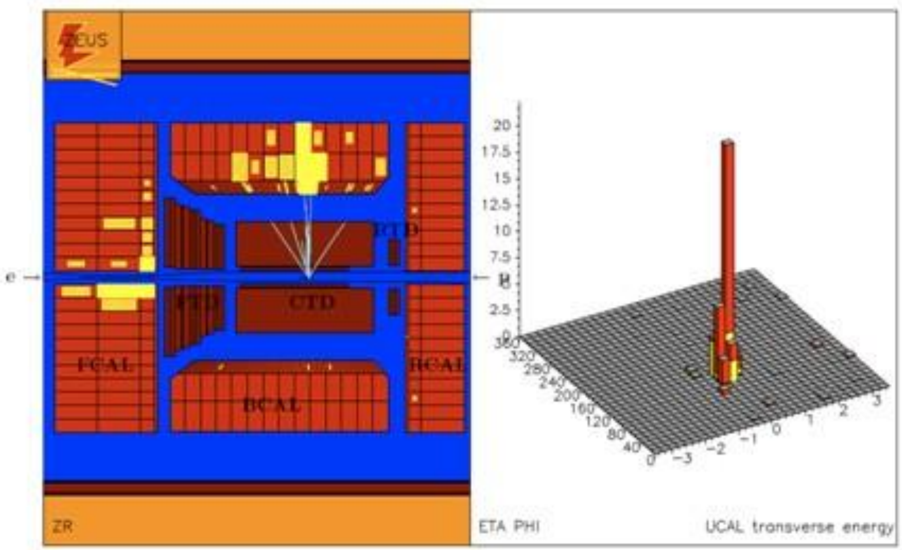
Note that the total area under the curve measures the momentum fraction in the nucleon carried by quarks, $P_q \sim 0.5$. The missing momentum fraction is ascribed to gluons, which cannot be seen directly in DIS experiments

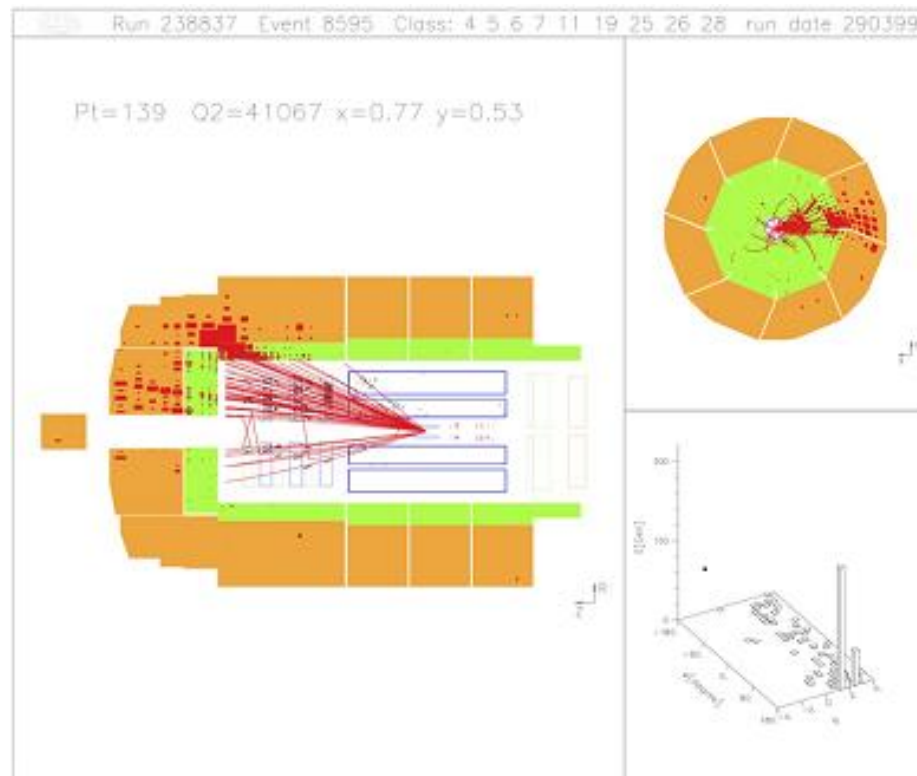
CC e + N $\rightarrow \nu_e$ + X Scattering

At very high energies,
in ep scattering also a W boson can be exchanged instead of a γ (or Z boson) $\rightarrow$ CC DIS.
Only a hadronic jet will be observed
(the outgoing ν is undetected)



Hera ep collider (Zeus + H1)





Sum Rules

Various predictions can be made for integrals – sum rules – over quark densities, which have simple interpretations in the QPM.
These sum rules are predictions based on QCD, therefore a powerful test of QCD.

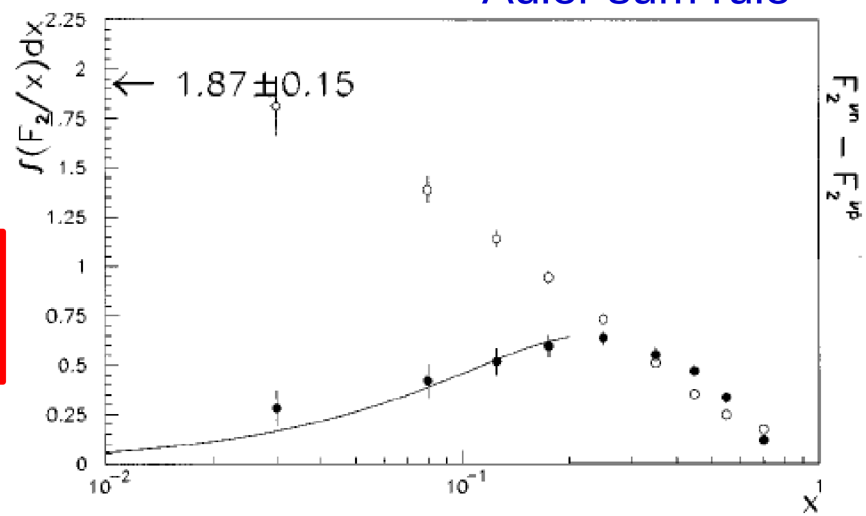
There are 2 such sum rules for ν interactions
(we already encountered the Gottfried sum rule comparing p and n quark distributions):

the Adler sum rule

$$I_A = \int_0^1 \frac{F_2^{\bar{\nu} p} - F_2^{\nu p}}{x} dx = \int_0^1 \frac{F_2^{\nu n} - F_2^{\nu p}}{x} dx = \int_0^1 2\left(u_V - d_V\right) dx = 2$$

this sum rule is Q^2 independent and hence has no QCD corrections

Adler sum rule



the Gross – Llewellyn Smith

$$I_{GLS} = \int_0^1 \frac{xF_3}{x} dx = \int_0^1 \left(u_V + d_V\right) dx = 3\left(1 - \frac{\alpha_S}{\pi}\right)$$

$I_{GLS} = 2.64 \pm 0.06$

to derive these sum rules use the definitions of F_2 and F_3 in terms of quark densities

For Next Week

Study the material and prepare / ask questions
Study ch. 12 (sec. 7 to 10) and ch. 13 (sec. 5) in Halzen & Martin
and / or ch. 12 (sec. 2 to 5) in Thomson

Do the homeworks

Next week we will study the Electroweak unification

have a first look at the lecture notes, you can already have questions
read ch. 13 (sec. 1 to 7) in Halzen & Martin
and / or ch. 15, ch. 16, and app D in Thomson