Advanced Particle Physics 2

Strong Interactions
and Weak Interactions

L12 – The Higgs Mechanism

(http://dpnc.unige.ch/~bravar/PPA2/L12)

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Massive Gauge Bosons

1. massive gauge bosons break local gauge invariance of QED, QCD and EW theory

Consider the QED Lagrangian with a massive photon (add the mass term $m^2 A_\mu A^\mu$)

$$\boxed{L_{QED} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m_e\right)\psi - q\bar{\psi}\gamma^\mu A_\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A^\mu A_\mu}$$

Under local U(1) gauge transformations, the fermion field Ψ and the photon field A_μ transform as

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x) \qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$$

while the photon mass term transforms as

$$\boxed{\frac{1}{2}m_\gamma^2 A^\mu A_\mu \to \frac{1}{2}m_\gamma^2\left(A^\mu - \frac{1}{q}\partial^\mu \alpha\right)\left(A_\mu - \frac{1}{q}\partial_\mu \alpha\right) \neq \frac{1}{2}m_\gamma^2 A^\mu A_\mu}$$

which is explicitly non invariant.

2. massive fermions break local gauge invariance of EW theory (not QED nor QCD)

Likewise the fermion mass term

$$\boxed{-m\bar{\psi}\psi = -m\bar{\psi}\left[\frac{1}{2}\left(1-\gamma^5\right) + \frac{1}{2}\left(1+\gamma^5\right)\right]\psi = -m\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)}$$

mixes terms with different chiralities and is not invariant under $SU(2)_L \times U(1)_Y$ transformations because of different transformation properties of the fermion fields ψ_L and ψ_R. This mass term, however, is invariant under QED U(1) and QCD SU(3) local gauge transformations because the ψ_L and ψ_R fields transform in the same way.

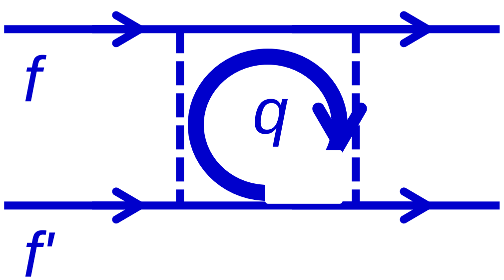
Massive Gauge Bosons

Why insist on rejecting terms like $M^2 W_\mu W^\mu$?

$\Rightarrow$ the theory is not renormalizable and loses all predictive power

(or each order in the perturbative expansion one has to introduce cutoffs specific to the order)

Consider for instance a higher order contribution to the scattering of two fermions.



To calculate the amplitude integrate over all propagators (loop)

$$M \sim \int \mathrm{d}^4 q \ (\text{propagators})$$

All possible 4-momenta *q* appear in the amplitude (internal lines), the momentum transfer q is not limited from above.

In QED, this box diagram represents the exchange of 2 photons (vacuum polarization). The propagator $ig_{\mu\nu}/q^2$ makes the integral finite (all orders renormalized with e_R and m_R).

In the case the boson is massive, the propagator yields a divergent integral for large 4-momenta *q*.

$$-i\frac{g_{\mu\nu} - q_\mu q_\nu / M^2}{q^2 - M^2} \xrightarrow[q^2 \to \infty]{} i\frac{q_\mu q_\nu}{q^2 M^2}$$

Introduce a cutoff on *q*? New parameter in the theory!

For each new diagram a new set of cutoff parameters are required and the theory cannot be renormalized because of too many cutoff parameters.

$\Rightarrow$ Any theory breaking local gauge invariance is not renormalizable!

The Higgs Mechanism

Gauge invariance can coexists with massive boson fields, if the mass of these fields is in reality an artefact, a *fake* conclusions that we infer from their sub-luminal propagation. There are different possibilities how to realize the sub-luminal propagation of a particle, which in reality is massless (a massless particle propagates at the speed of light).
We will study the spontaneous symmetry braking or the Higgs mechanism.

The Higgs mechanism provides a mechanism for W^+, Z^0, and W^- gauge bosons to acquire mass without violating the local gauge invariance principle.

We will arrive at this result in several steps:

1. symmetry breaking for a real scalar theory (discrete symmetry)
2. symmetry breaking for a complex scalar theory (continuous symmetry)
3. symmetry breaking in a local gauge theory (U(1) local symmetry)
4. symmetry breaking in a local SU(2) gauge theory
5. symmetry breaking of the electroweak gauge group $SU(2)_L \times U(1)_Y$

At the same time, fermions acquire mass via Yukawa couplings to the same Higgs field.

With the discovery of the Higgs boson in 2012 (M_H ~ 126 GeV) the Standard Model of particle physics is complete.

The Higgs Mechanism

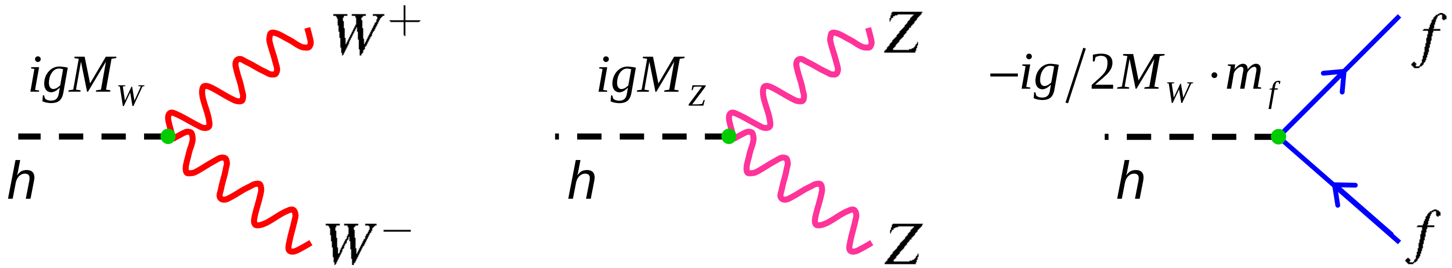
Propose a scalar spin-0 field *h*(*x*) that permeates the space-time (i.e. it is everywhere) with a non-zero vacuum expectation value (VEV) $\langle 0|h(x)|0\rangle \neq 0$.

But what is the vacuum?

1) state of lowest energy (otherwise the system can always evolve to that state)
2) state with no fields $\phi = 0$ (VEV $=\langle 0|\phi|0\rangle = 0$) $\rightarrow$ state with fields $\phi \neq 0$ (VEV $\neq 0$)

Gauge bosons propagating through the vacuum with a non-zero Higgs VEV correspond to massive particles.

- the Higgs boson is electrically neutral but carries weak isospin T = ½ and T_3 = -1/2, and weak hypercharge Y = 1
- the $W^{\pm}$ and Z bosons couple to the weak isospin and hypercharge and acquire mass
- the photon does not couple to the Higgs field and stays massless
- the Higgs mechanism results in absolute predictions for the masses of gauge bosons
- fermion masses are also ascribed to interactions with the Higgs field, however no predictions of masses are made (Higgs boson couplings are proportional to their mass)



Real Scalar Field

We start with a world containing only a real scalar field $\phi(x)$, without fermions, without matter, characterized by the Lagrangian density

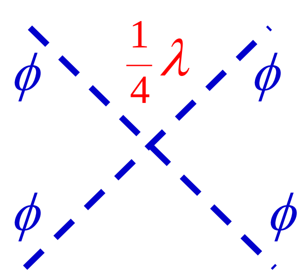
$$\boxed{L_\phi = T\left(\partial_\mu \varphi\right) - V\left(\phi\right)}$$

in which T is the kinetic term (it depends only on the components of the field gradient), while V is a potential function. For the “usual” free scalar field, $V(\phi)$ is a second-order function in ϕ, $V = m^2/2\ \phi^2$. The “zero” solution corresponds to the minimum of the potential function, and the quantization of the free scalar field corresponds to the excitations (small oscillations) around the equilibrium point at $\phi = 0$.

The Lagrangian is symmetric w.r.t. the reflection transformation $\phi \to -\phi$, and its lower state $\phi = 0$ is symmetric and stable.

Now, let’s introduction into $V(\phi)$ a positive term of the form ϕ^4 describing self-interactions of the field (it can also be seen as an expansion of $V(\phi)$ around $\phi = 0$):

$$\boxed{V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4}$$



and the Lagrangian becomes

$$\boxed{L_\phi = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)}$$

where we have introduced the term $\frac{1}{4}\lambda\phi^4$, which describes the self-interactions among the field ϕ, with constants $\mu^2, \lambda > 0$. This Lagrangian is symmetric for $\phi \to -\phi$.

Comparison with the Lagrangian for the free scalar fields of mass *m* (Klein-Gordon Lagrangian)

$$\boxed{L_{KG} = \frac{1}{2}\partial_\mu \varphi\, \partial^\mu \varphi - \frac{1}{2} m^2 \phi^2}$$

shows that L_ϕ describes a field of mass μ (provided $\mu^2 > 0$), which interacts with itself via a quartic vertex with a coupling λ (ϕ – 4 theory).

The potential density $\boxed{V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4}$

has a minimum for $\phi = 0$, $\left.\frac{\mathrm{d}V}{\mathrm{d}\phi}\right|_{\phi=0} = 0$ and $\left.\frac{\mathrm{d}^2V}{\mathrm{d}\phi^2}\right|_{\phi=0} = \mu^2 > 0$

with the vacuum expectation value (VEV) $\boxed{\langle 0|\phi|0\rangle = 0}$.

That is expected, since the potential energy should have a minimum in absence of fields. Note that there are no odd power terms, otherwise *V* would have no minimum.

The interpretation changes dramatically, if $\mu^2 < 0$ ($\rightarrow$ imaginary mass).
The term proportional to ϕ^2 cannot be interpreted as a mass term, since the mass is an observable and must be real. This may suggest the introduction of supraluminal particles with imaginary mass (tachions). To us, however, what is important, is that the point $\phi = 0$ in not a minimum of the potential V.
This Lagrangian, however, is still symmetric for $\phi \rightarrow -\phi$.

The potential has a local maximum for $\phi = 0$, because $\left.\frac{dV}{d\phi}\right|_{\phi=0} = 0$ and $\left.\frac{d^2V}{d\phi^2}\right|_{\phi=0} < 0$.

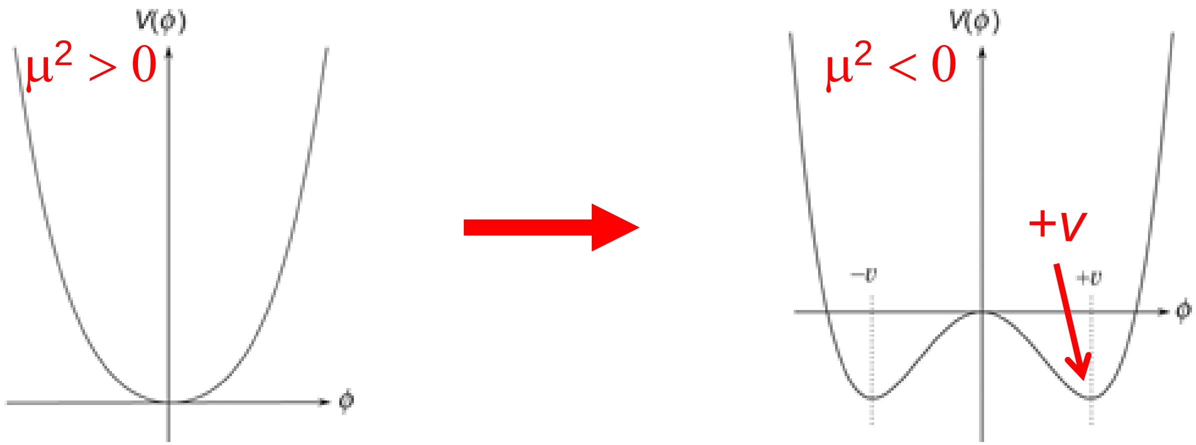
Local minima appears for $\mu^2 + \lambda\phi^2 = 0$ in two points (stable equilibrium) , i.e.

$$\boxed{\phi_{\min} = \pm v = \sqrt{\frac{-\mu^2}{\lambda}}}$$

with v, the vacuum expectation value VEV of the field

$$\boxed{\langle 0|\phi|0\rangle = \phi_0 = \pm\sqrt{\frac{-\mu^2}{\lambda}} = \pm v \neq 0}$$

The VEV is therefore not zero. We call vacuum the state of minimal energy.



Both minima have the same energy

$$\boxed{V(\pm\phi_0) = V_0 = -\mu^4/4\lambda}$$

as a result of which the corresponding system has a doubly-degenerate vacuum state.

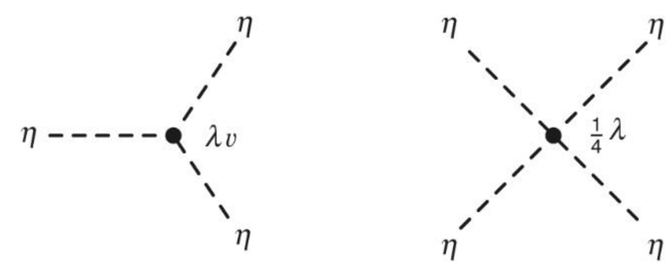
To study excitations of the field around the ground sate with this Lagrangian ($\mu^2 < 0$ and $\lambda > 0$), we must develop the field around a minimum of the energy E_{min}.
In this case the minimum corresponds to a finite value of the field ϕ: $\phi_{min} = +v$ or $\phi_{min} = -v$.
The vacuum, therefore, does not correspond to a situation with no fields, but it contains a field $\phi(x) = \pm v$ for all the space-time points x. We have to choose a minimum among two possibilities, $+v$ and $-v$, we choose $+v$ (arbitrary decision).

To perform the expansion of the field around E_{min} we transform ϕ (shift ϕ to the minimum) as

$$\boxed{\phi(x) = +v + \eta(x)}$$

The Lagrangian can be rewritten in terms of the field $\eta(x)$ as

$$\boxed{L_\eta = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda v^4}$$



L_η is not invariant for $\eta \to -\eta$ (the term $\lambda v\eta^3$ changes sign): the symmetry is broken.
The coefficient λv^2 of the η^2 term is now positive and comparison with the Klein-Gordon Lagrangian allows us to interpret this term as a real mass term

$$\boxed{m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0}$$

note $\langle 0|\eta|0\rangle = 0$

The Lagrangian can be finally rewritten as

$$\boxed{L_\eta = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \frac{1}{2}m_\eta^2\eta^2 - V(\eta)}$$

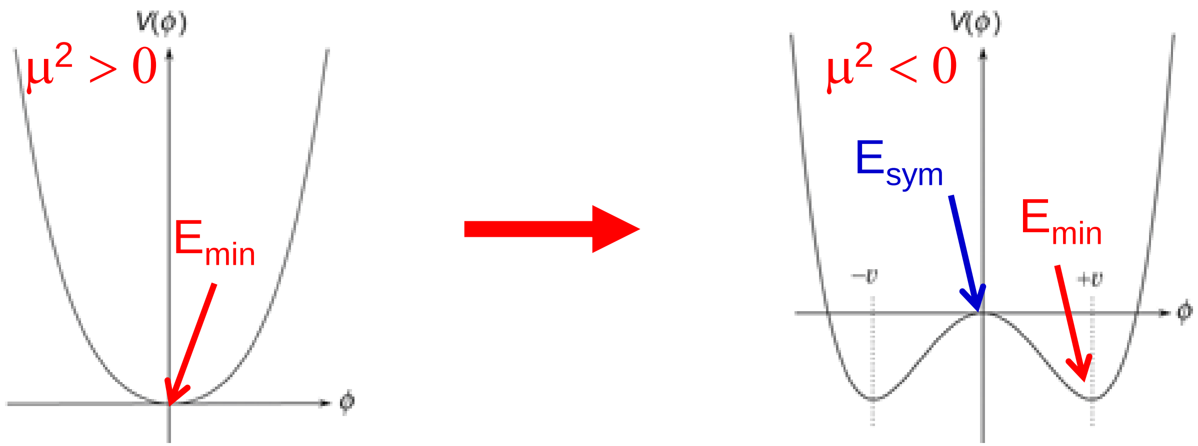
with

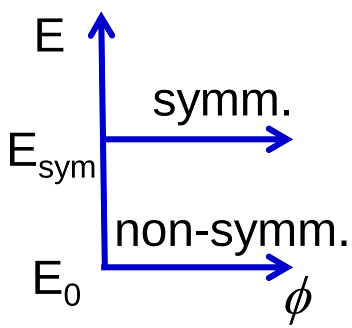
$$\boxed{V(\eta) = \lambda v^2\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda v^4}$$

The potential V describes triple and quartic self-interactions among the fields η.

Spontaneous Symmetry Breaking

1. Find a non-symmetric solution of a theory which is otherwise symmetric.
2. The symmetric state does not coincide with the ground state.
3. Identify a mechanism that evolves the system
   from the symmetric state to the non-symmetric state.





$\boxed{\langle 0|\phi|0\rangle = 0}$ $\boxed{\langle 0|\phi|0\rangle \neq 0}$

With the transformation $\phi \to \eta$ we have lost the symmetry of the initial Lagrangian.
L_η is not invariant for $\eta \to -\eta$ (the term $\lambda v \eta^3$ changes sign) $\Rightarrow$ different physics.
This is due to the fact that we choose the minimum at +v. We could have also chosen the vacuum at –v, but not both at the same time. Our choice has broken the symmetry of the system, and this symmetry breaking has given mass to the field η(x).

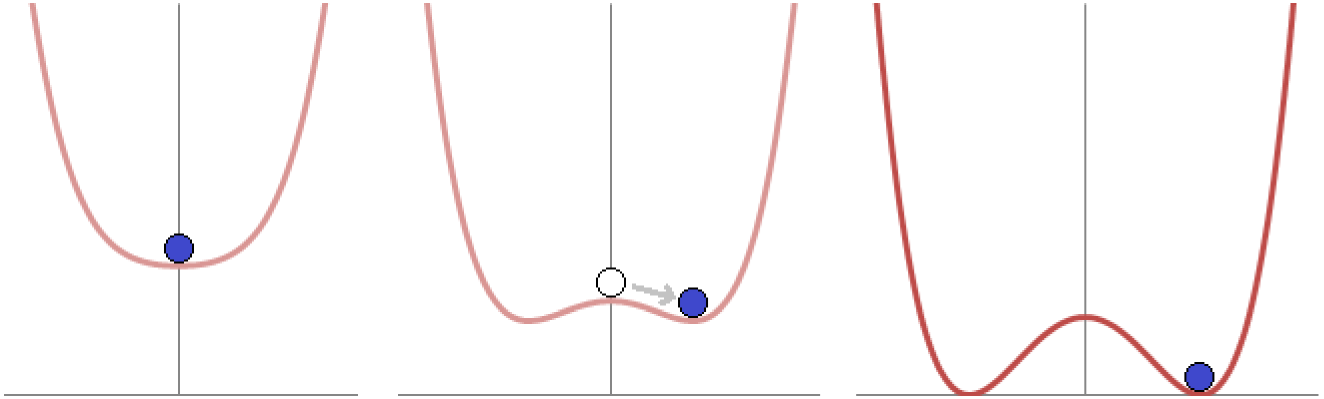
The choice of the ground state among various possibilities, which are linked by a symmetry is what we call spontaneous symmetry breaking.
The symmetry of the system is no longer evident in its ground state; more precisely, we should speak of hidden symmetry, since the system does not exhibit in its ground state the symmetry, which is well visible in the excited states of the system.

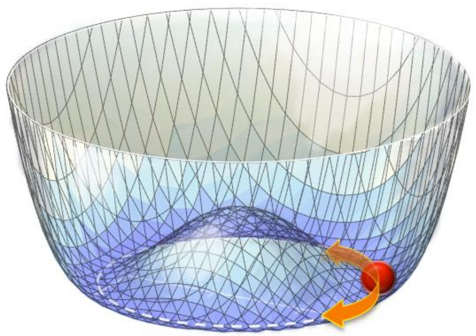
One can ask how such a trivial transformation, which leaves the Lagrangian intact, can transform the massless field ϕ into the massive filed η.
With an exact calculation one would find perfectly equivalent solutions with L_ϕ and L_η .
We are not seeking, however, an exact solution, but we approximate the solution with a perturbative expansion. The perturbative expansion is possible only for L_η around a minimum of the potential energy $\eta = 0$ (i.e. $\phi = v$).
Both Lagrangians describe a massive particle, but we know how to calculate only with L_η.

Physical Analogies

In Nature, many systems exhibits spontaneous symmetry breaking. These systems are described by some *functions* (Lagrangians, Hamiltonians, forces, …) possessing some symmetry, while the real physical state of the system corresponding to a particular solution of the equation of motion does not exibit this symmetry.
This arises when the lowest symmetric state does not have the lowest possible energy and it is itself unstable (see e.g. magnetization). The actual cause of symmetry breaking may turn out to be an infinitesimal non-symmetric perturbation.

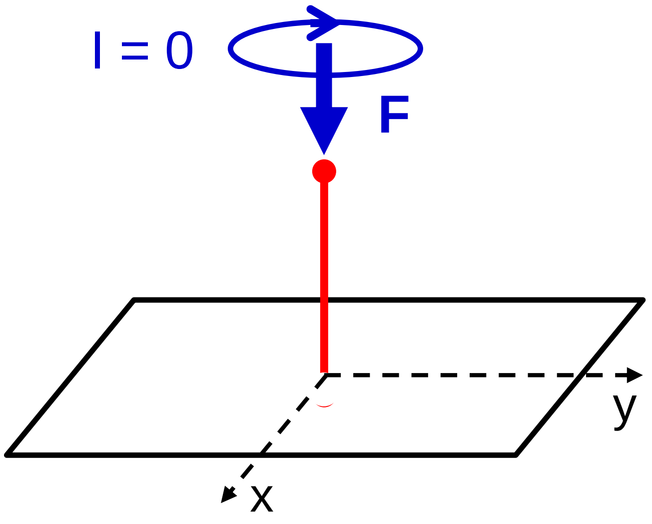
A simple example is provided by the bottom of a bottle and a small ball inside it.
The bottle has rotational symmetry around the vertical axis.
Let the ball fall along this axis.
When the ball reaches the bottom, the ball will not rest on the central protuberance but will roll down to the periphery.
The final state has lost its initial rotational symmetry and the ball has acquired an angular momentum *L* around the symmetry axis.





Mechanical Analogy

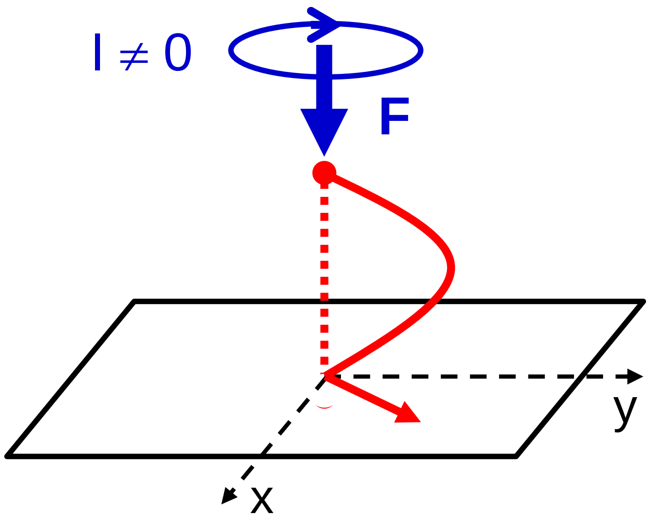
The following is usually presented as a mechanical example to develop an appreciation of this surprising phenomenon of symmetry breaking.



Imagine a one dimensional needle. Around its axis, the needle has no angular momentum.

Then press the needle with a vertical force larger than the elasticity limit of the needle.

Initially nothing happens, the system remains in an equilibrium state (well, you will just feel pain …).



Starting from a quantum fluctuation, the lattice of the needle weakens in an unpredictable point, and the needle bends in a random direction.

The cylindrical symmetry of the system has disappeared spontaneously.
At the same time an angular momentum around the initial symmetry axis has appeared.
The system has acquired a kind of mass.

Electromagnetic Analogy

Gauge symmetry assures that a Quantum Field Theory gives finite results and requires massless fields. However real fields can be massive: it is possible that the quanta of the fields do not have a "real" mass, but that their free propagation is slowed down by constraints connected to the space-time.

Consider for instance the electromagnetic radiation propagating through a plasma of electrons. Because the plasma acts as a polarisable medium → "dispersion relation"

$$n^2 = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

with ω_p the plasma cutoff-frequency.

Electromagnetic waves propagate in the plasma only for frequencies $\omega > \omega_p$:

$$\boxed{E = \hbar\omega \quad > \quad E_p = \hbar\omega_p}$$

Above this frequency EM waves propagate with a group velocity v_g

$$\boxed{v_g = cn = c\sqrt{1 - E_p^2 / E^2}}$$

Rearranging gives

$$\boxed{\frac{E_p^2}{E^2} = 1 - \frac{v_g^2}{c^2} \quad \Rightarrow \quad E = E_p \Big/ \sqrt{1 - v_g^2/c^2} = \gamma m c^2} \quad \text{with} \quad m = E_p / c^2$$

Massless photons, which propagate through a plasma, behave as massive particles propagating in a vacuum with velocity $v_g < c$!

Complex Scalar Fields

A quite simple but profound generalization arises in the transition from the single-component field to an isotopic multiplet $\phi = (\phi_1, \phi_2, \ldots \phi_N)$. For simplicity, let's consider a scalar complex field

$$\boxed{\phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)}$$

where ϕ_1 and ϕ_2 are two spin-0 real fields, with the following Lagrangian density

$$\boxed{L_\phi = \left(\partial_\mu \varphi\right)^* \left(\partial^\mu \phi\right) - \mu^2 \phi^* \phi - \lambda\left(\phi^* \phi\right)^2}$$

and $\mu^2 < 0$, i.e. without a real mass term and $\lambda > 0$.

The discrete reflection symmetry is replaced by the continuous symmetry of isotopic rotations, i.e. L_ϕ is invariant under global phase transformations

$$\boxed{\phi(x) \to \phi'(x) = e^{i\alpha}\phi(x)}$$

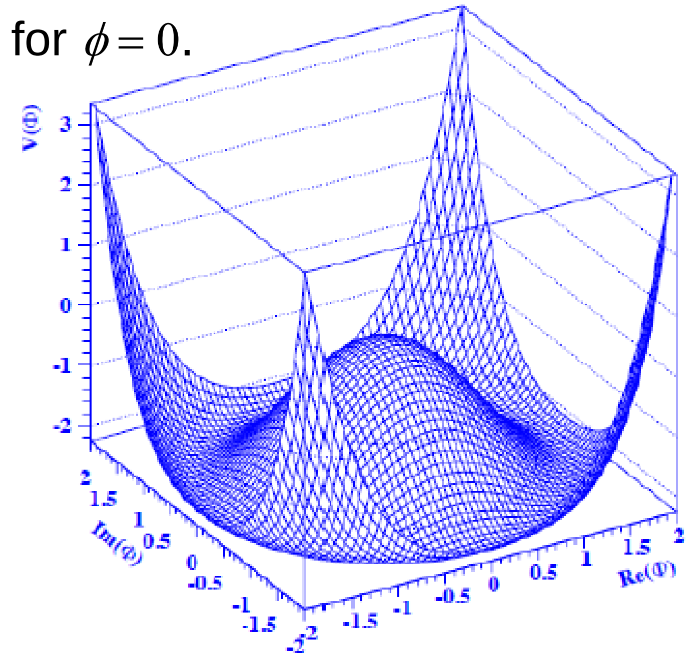
with α a real constant.

This Lagrangian is invariant under global phase transformations of the U(1) group.

Rewritten in terms of the ϕ_1 and ϕ_2 fields, the Lagrangian becomes

$$\boxed{L_\phi = \frac{1}{2}\partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{1}{2}\partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{1}{2}\mu^2\left(\phi_1^2 + \phi_2^2\right) - \frac{1}{4}\lambda\left(\phi_1^2 + \phi_2^2\right)^2}$$

As before, the potential has a local, unstable, maximum for $\phi = 0$.
Local minima occur for

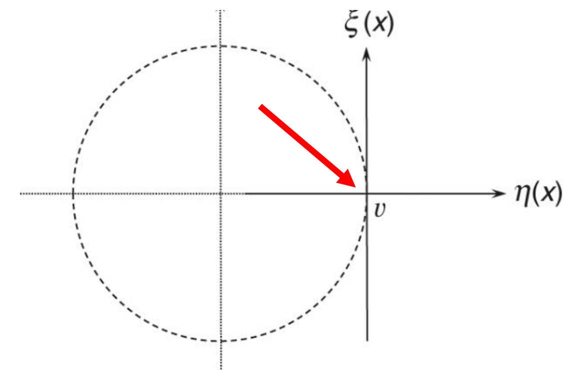


$$\boxed{\phi^\dagger \phi = \phi_1^2 + \phi_2^2 = -\frac{\mu^2}{\lambda} = v^2 > 0}$$

This is the equation of a circle in the $\phi_1 - \phi_2$ plane.

Now let's shift the field ϕ by a constant vector $\phi_0 = v$, satisfying $v^2 = -\mu^2 / \lambda$.
We have the freedom to choose any minimum of the potential for our (perturbative) vacuum.
Without loss of generality we choose $\phi_1 = v$ and $\phi_2 = 0$, such that

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\xi(x)\right)}$$

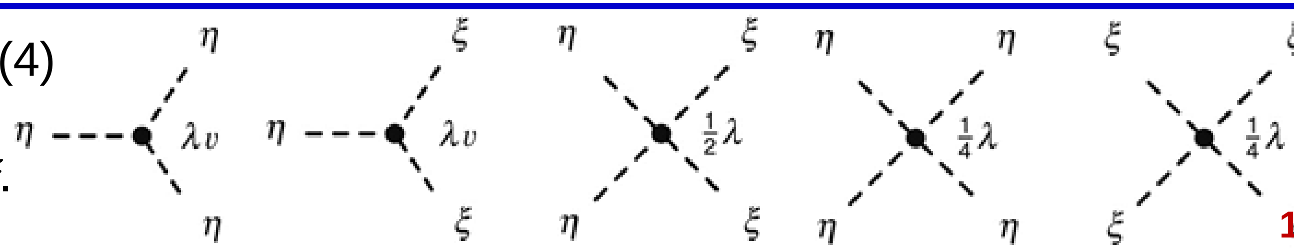


where we introduced two new local fields η(x) and ξ(x).

Inserting this ϕ(x) into the Lagrangian gives

$$\boxed{L' = \frac{1}{2}\partial_\mu \xi \, \partial^\mu \xi + \frac{1}{2}\partial_\mu \eta \, \partial^\mu \eta - \lambda v^2 \eta^2 - \lambda v \eta^3 - \lambda v \eta \xi^2 - \frac{1}{2}\lambda \eta^2 \xi^2 - \frac{1}{4}\lambda \eta^4 - \frac{1}{4}\lambda \xi^4 + \frac{1}{4}\lambda v^4}$$

The terms o ~ (3) and o ~ (4) describe (self)interactions between the fields η and ξ.



Goldstone Theorem

The resulting Lagrangian is not invariant under global phase transformations of the η and ξ fields, because we have chosen a particular ground state.
We can recognize a mass term for the field η with the correct sign

$$\boxed{-\lambda v^2\eta^2 = +\mu^2\eta^2 = -\frac{1}{2}m_\eta^2\eta^2}$$

corresponding to a real mass $m_\eta = \sqrt{-2\mu^2}$ (recall $\mu^2 < 0$): the shifted component has acquired a mass. There is no mass term for the field ξ, even though L' contains the kinetic term $1/2\left(\partial_\mu \xi\right)^2$ (there is no potential in the ξ "direction": the other component corresponds to massless degrees of freedom).
The theory contains a massless scalar field $\xi(x)$, referred to as Goldstone boson.

This is an example of the Goldstone theorem, which predicts the appearance of a massless scalar field whenever we spontaneously break the continuous symmetry of a system. This property is invariably associated with the spontaneous breakdown of a continuous symmetry, because each of the massless degrees of freedom corresponds to an infinite degree of degeneracy of the vacuum state.
(virtual motion of the ball within the circular hollow at the bottom of the bottle)

In summary, the field ϕ (field η) has acquired the mass $m_\eta = \sqrt{2\lambda v^2}$,
but also a new massless scalar particle ξ has appeared.

Symmetry Breaking In a Local Gauge Theory

It seems that we are moving in the wrong direction:
by giving mass to a field ϕ by spontaneously breaking the symmetry of the system, we generated a Goldstone boson that we did not foresee.
Let's study the symmetry breaking of a local gauge theory of the U(1) type:

$$\boxed{\phi \to e^{i\alpha(x)}\phi}$$

To realize it in the Lagrangian we replace the ∂_μ derivative by the covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu(x)$$

with the gauge fields A_μ transforming as

$$A_\mu(x) \to A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$$

The gauge invariant Lagrangian becomes

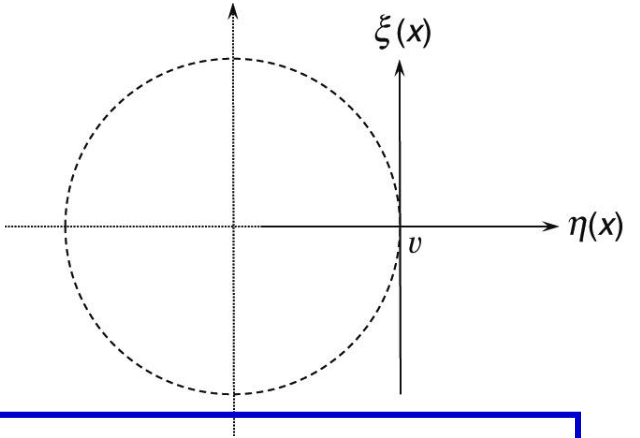
$$\boxed{L = \left(\partial_\mu - iqA_\mu\right)\phi^*\left(\partial^\mu + iqA^\mu\right)\phi - \mu^2\phi^*\phi - \lambda\left(\phi^*\phi\right)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

For $\mu^2 > 0$ this resembles the QED Lagrangian for a scalar particle of mass μ and charge *q*, enlarged by a self-interaction of the field ϕ.

I'm using the same notations $A_\mu(x)$ and *q* to indicate the associated gauge field and its coupling, but this is not QED!
We are working with scalars and a Lagrangian of the Klein-Gordon type.

However, for $\mu^2 < 0$, the term $\mu^2\phi^*\phi$, as before, cannot be interpreted as mass term for the field ϕ.
For the ground state we choose (as before)

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\xi(x)\right)}$$



After substitution, the Lagrangian becomes

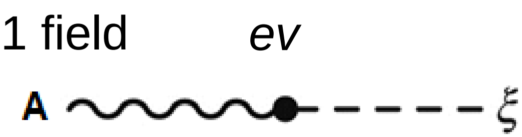
$$\boxed{L' = \frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - v^2\lambda\eta^2 + \frac{1}{2}q^2v^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + qvA_\mu\partial^\mu\xi + WW\left(\xi,\eta\right)}$$

We can identify the mass terms for the fields η(x) (Klein-Gordon eq. –sign) and A_μ(x) (Proca eq. +sign), but not for ξ(x):

$$\boxed{m_\eta = \sqrt{2\lambda}v \qquad m_A = qv \qquad m_\xi = 0}$$

We succeeded in generating dynamically a mass for the gauge field A_μ, but we are not yet done because of the massless Goldstone boson ξ.

By giving mass to A_μ(x) we gained 1 d.o.f.! i.e. one more polarization state for A_μ(x) (3 spin components instead of 2).
Moreover, the term $A_\mu\partial^\mu\xi$ describes the interaction of a spin-1 field with a spin-0 field, in which the spin "disappears" and such terms cannot exist.



We have the freedom to make a gauge transformation to eliminate the unphysical d.o.f. or the Goldstone field ξ(x) by choosing the phase α(x) for the ground state.

Let's group the terms in L' involving the Goldstone field $\xi(x)$ and $A_\mu(x)$

$$\boxed{\frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi+\frac{1}{2}q^2v^2A_\mu A^\mu+qvA_\mu\partial^\mu\xi=\frac{1}{2}q^2v^2\left[A_\mu+\frac{1}{qv}\partial^\mu\xi\right]^2}$$

and apply the following gauge transformation on A_μ

$$\boxed{A_\mu(x)\to A_\mu{}'(x)=A_\mu(x)+\frac{1}{qv}\partial_\mu\xi(x)}$$

Since the starting Lagrangian was constructed to be invariant under local U(1) gauge transformations, the physical predictions are unchanged.

With this A' (x) the transformed L' becomes

$$\boxed{L'=\frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta-v^2\lambda\eta^2+\frac{1}{2}q^2v^2A_\mu{}'A^{\mu\prime}-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}+WW\left(\eta,A_\mu\right)}$$

With the appropriate choice of the gauge,

the Goldstone field $\xi(x)$ no longer appears in the Lagrangian.

This choice of gauge corresponds to taking $\alpha(x) = -\xi(x)\,/\,v$, i.e. a particular value for the phase $\alpha(x)$ corresponds to a particular choice for the filed $\xi(x)$,

with $\phi(x)$ transforming as

$$\boxed{\phi(x)\to\phi'(x)=e^{-i\xi(x)/v}\phi(x)}$$

We expanded the field $\phi(x)$ around the physical vacuum as

$$\boxed{\phi(x)=\frac{1}{\sqrt{2}}\left(v+\eta(x)+i\xi(x)\right)\approx\frac{1}{\sqrt{2}}\left(v+\eta(x)\right)e^{i\xi(x)/v}}$$

The effect of the gauge transformation on $\phi(x)$ is then

$$\boxed{\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}} e^{-i\xi(x)/v}\left(v+\eta(x)\right)e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}\left(v+\eta(x)\right)}$$

i.e. $\phi(x)$ is real.

The gauge in which the Goldstone field $\xi(x)$ is eliminated from the Lagrangian corresponds to choosing the complex scalar field $\phi(x)$ real (unitary gauge).

(with different notation) We choose a ground state with the fields $h(x)$, $\theta(x)$, and $A_\mu(x)$, all real (also set e = q), such that

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}\left(v+h(x)\right)e^{+i\theta(x)/v}$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{ev}\partial_\mu \theta(x)$$

with $\alpha(x) = -\theta(x)/v$.

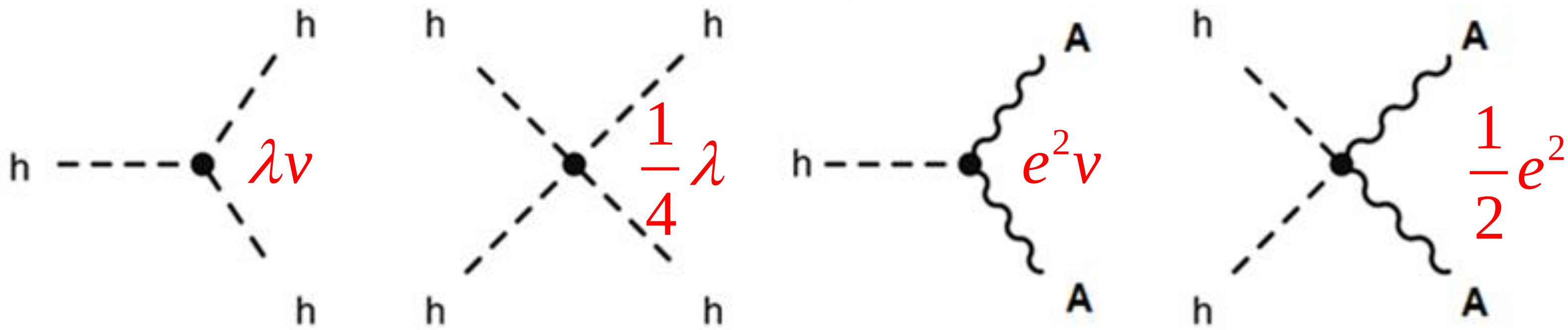
With these substitutions, the Lagrangian becomes (the constant term $\lambda v^4/4$ is left out)

$$\boxed{L' = \underbrace{\frac{1}{2}\partial_\mu h\,\partial^\mu h - \lambda v^2 h^2}_{h\text{ field}} + \underbrace{\frac{1}{2}e^2v^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{A_\mu\text{ field}} + ve^2 A_\mu A^\mu h + \frac{1}{2}e^2 A_\mu A^\mu h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4}$$

$M_H = \sqrt{2\lambda}\,v$ $\qquad$ $M_A = ev$

The field $\theta(x)$ has disappeared in the gauged Lagrangian and the Goldstone boson has been reabsorbed. The new Lagrangian describes the massive scalar Higgs field $h(x)$ and the massive gauge field $A_\mu(x)$.

The Higgs field interacts with itself and with $A_\mu(x)$:



The mass of the gauge boson A_μ

$$M_A = ev$$

is completely determined by the strength of the coupling e and the VEV v of the Higgs field.

The mass of the Higgs boson is given by

$$M_H = \sqrt{2\lambda}v$$

The VEV v

$$\mathrm{VEV} = \langle 0 | h(x) | 0 \rangle = 246\ \mathrm{GeV}$$

sets the scale for the masses of both the gauge boson and Higgs boson.

This is the Higgs mechanism.

We have introduced two new parameters in the Standard Model:
the vacuum expectation value v and the mass of the Higgs boson M_H.

Local SU(2) Gauge Invariance

Let's consider now an SU(2) complex field doublet (no spin, 4 real scalar fields)

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix}}$$

The resulting Lagrangian is invariant under global phase transformations of the SU(2) symmetry group

$$\phi(x) \to \phi'(x) = e^{i\alpha_a \tau^a/2}\phi(x)$$

with α_a a 3 component vector in SU(2) space and τ_a the Pauli matrices that generate the transformation.
When requiring local gauge invariance

$$\boxed{\phi(x) \to \phi'(x) = e^{i\alpha_a(x)T^a}\phi(x)}$$

we have to allow the complex fields to interact with "vector" gauge fields $W_\mu^a(x)$ introduced via the covariant derivative (see L3 and L11)

$$\partial_\mu \to D_\mu = \partial_\mu + igT_a W_\mu^a(x)$$

and the $W_\mu^a(x)$ fields transforming as

$$W_\mu^a(x) \to W_\mu'^a(x) - \frac{1}{g}\partial_\mu \alpha^a(x) - i\,\varepsilon_{abc}\,\alpha^b(x)\,W_\mu^c(x)$$

g is the coupling constant of the fields $\phi(x)$ with the gauge fields $W_\mu^a(x)$.

The resulting gauge invariant Lagrangian describing the complex field doublet $\phi(x)$ is given by

$$\boxed{L = \left(\partial_\mu \phi + ig\frac{\tau_a}{2}W_\mu^a\phi\right)^\dagger \left(\partial^\mu \phi + ig\frac{\tau^a}{2}W_a^\mu\phi\right) - V(\phi) - \frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu}}$$

with the potential energy term $V(\phi)$ (we have chosen this potential!)

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

and the kinetic term for the 3 *W* gauge fields expressed in terms of field tensors

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon^{abc}W_\mu^b W_\nu^c$$

The “vector products” in the field transformations and the tensors appear because the SU(2) group is non Abelian and the generators do not commute.

For $\mu^2 > 0$ this Lagrangian describes 4 scalar particles, ϕ_1, ϕ_2, ϕ_3, and ϕ_4, with mass μ, invariant under local SU(2) transformations,
that interact among themselves $\lambda\left(\phi^\dagger\phi\right)^2$ and with the 3 gauge fields W_μ^1, W_μ^2, and W_μ^3.
The gauge fields interact also between themselves.

Local SU(2) Symmetry Breaking

Again, we are interested in the case $\mu^2 < 0$ and $\lambda > 0$, because it will allow us to spontaneously break the symmetry of the system.
The potential $V(\phi)$ has an infinite set of degenerate minima for

$$\boxed{\phi^\dagger\phi = \frac{1}{2}\left(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2\right) = -\frac{\mu^2}{2\lambda}}$$

The ensemble of these minima is invariant under SU(2) transformations of the fields ϕ.
To study the excitation of the fields around a vacuum point (minimum of energy) we choose a specific minimum and the symmetry of the system will not be manifest anymore.
We choose for the ground state

$$\boxed{\phi_1 = \phi_2 = \phi_4 = 0 \qquad \phi_3^2 = -\frac{\mu^2}{\lambda} = v^2}$$

The fields can be expanded around this minimum by posing

$$\boxed{\phi(x) = \frac{1}{2}\begin{pmatrix} \phi_1(x)+i\phi_2(x) \\ v+\eta(x)+i\phi_4(x) \end{pmatrix}}$$

Instead of repeating the derivation as for the U(1) symmetry breaking case and gauge away the Goldstone fields, the ϕ(x) doublet is already written in the unitary gauge (i.e. we choose it real)

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}}$$

with the vacuum state

$$\boxed{\phi_0(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}}$$

By this choice, the SU(2) symmetry is not manifest anymore and we say that the symmetry has been spontaneously broken.

Thanks to the gauge invariance, after a (small) rotation is SU(2) we recover the initial form of the field $\phi(x)$:
[the Lagrangian is invariant under gauge transformations by construction]

$$\phi(x) = e^{iT_a\theta^a(x)/v}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1+i\theta_3/v & (i\theta_1+\theta_2)/v \\ (i\theta_1-\theta_2)/v & 1-i\theta_3/v \end{pmatrix}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \theta_2+i\theta_1 \\ v+h-i\theta_3 \end{pmatrix}$$

All 4 fields can be described in terms of the field $h(x)$.
With our choice of the vacuum state, only the Higgs field $h(x)$ remains.

In the unitary gauge the Lagrangian does not contain the fields θ_1, θ_2, *and* θ_3 .
In other words we can absorb 3 out of 4 fields, θ_1, θ_2, and θ_3, giving mass to the 3 gauge bosons. The remaining field is the Higgs field. With an appropriate gauge transformation (unitary gauge) no Goldstone boson appears.

To obtain the mass term for the W fields, insert the vacuum state $\phi_0(x)$ in the Lagrangian

$$\boxed{L = \left(\partial_\mu \phi + igT_a W_\mu^a \phi\right)^\dagger \left(\partial^\mu \phi + igT^a W_a^\mu \phi\right) - V(\phi) - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu}}$$

The term proportional to W^2 (i.e. the mass term) is

$$\boxed{\left|igT_a W_\mu^a \phi_0\right|^2 = \frac{g^2}{8}\left|\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2 = \frac{g^2 v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(W_\mu^3\right)^2\right]}$$

Comparison with a typical mass term for massive vector bosons shows that

$$\boxed{M_W = \frac{1}{2} g v}$$

These 3 W_μ^i gauge fields have eaten up 3 Goldstone bosons and have become massive: the massive gauge bosons have 3 d.o.f. (longitudinal polarization) instead of 2. The Lagrangian describes 3 massive boson fields W_μ^i, vectors under the Lorentz group, and a scalar field $h(x)$, the Higgs boson.

In summary, in a theory that does not contain fermions we succeeded to transform the SU(2) locally invariant Lagrangian into a SU(2) locally invariant Lagrangian describing 3 massive vector bosons and one massive scalar boson via the spontaneous symmetry breaking. The massless Goldstone bosons, associated with the symmetry breaking, are not present thanks to the appropriate choice of a gauge transformation (ground state).

The Standard Model Higgs

Finally, we apply the Higgs mechanism to the Electroweak Lagrangian (L11):
We introduce 4 real scalar fields and an $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian

$$\boxed{L_H = \phi^\dagger \left(i\partial_\mu - gT_a W_\mu^a - g'\frac{Y}{2}B_\mu \right)\phi - V(\phi)}$$

with the usual potential with $\mu^2 < 0$ and $\lambda > 0$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi \right)^2$$

This additional term L_H is invariant under combined transformations of the $SU(2)_L \times U(1)_Y$ group provided that $\phi(x)$ belongs to a multiplet of the $SU(2)_L \times U(1)_Y$ group.
The minimal choice is to arrange the 4 fields in a weak isospin doublet T = ½ with weak hypercharge Y = 1 (Weinberg 1967)

$$\boxed{\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}}$$

With this minimal choice and the above Lagrangian we have specified the electro-weak sector of the Minimal Standard Model, i.e. the Glashow – Weinberg – Salam model.

We choose the following vacuum state $\phi_0(x)$

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \rightarrow \qquad \phi_{0,1} = \phi_{0,2} = \phi_{0,4} = 0 \qquad \phi_{0,3}^2 = -\frac{\mu^2}{\lambda} = v^2$$

with $v^2 = -\mu^2/\lambda$.

This ground state is characterized by T = 1/2, T_3 = – 1/2, and Y = 1,

The resulting Lagrangian is no longer invariant under $SU(2)_L$ or $U(1)_Y$ transformations.

On the other hand the electric charge of the field is $Q = T_3 + Y/2 = 0$,

and the system is symmetric under $U(1)_Q$ transformations.

The charge of the state $\phi_0(x)$ is $Q\phi_0 = 0$, and for each transformation (phase $\alpha(x)$)

$$\boxed{\phi_0(x) \rightarrow \phi_0'(x) = e^{i\alpha(x)Q}\phi_0(x) = \phi_0(x)}$$

This means that the $U(1)_Q$ symmetry is preserved and no mass is generated for the γ.

On the contrary, the $SU(2)_L$ and $U(1)_Y$ symmetries are broken and

the $W^\pm$ and Z^0 bosons acquire mass.

The mass term is obtained by inserting the ground state $\phi_0(x)$ in the Lagrangian L_H

$$\left|\left(-igT_aW_\mu^a - i\frac{g'}{2}B_\mu\right)\phi_0\right|^2 = \frac{1}{8}\left|\begin{pmatrix} gW_\mu^3 + g'B_\mu & g\left(W_\mu^1 - iW_\mu^2\right) \\ g\left(W_\mu^1 + iW_\mu^2\right) & -gW_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

$$= \frac{g^2v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right] + \frac{v^2}{8}\left(g'B_\mu - gW_\mu^3\right)\left(g'B^\mu - gW^{3,\mu}\right)$$

$$= \frac{g^2v^2}{4}W_\mu^+W^{-,\mu} + \frac{v^2}{8}\left(W_\mu^3, B_\mu\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix}$$

From the first term we deduce the mass of the $W^{\pm}$ bosons

$$\boxed{M_W = \frac{1}{2} g v}$$

The second term is non diagonal in the (W^3, B) basis

$$\frac{v^2}{8}\left(W_\mu^3, B_\mu\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix} = \frac{v^2}{8}\left(W_\mu^3, B_\mu\right)\mathrm{M}\begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix}$$

The “mass matrix” **M** can be diagonalized.

The masses of physical gauge bosons are given be the eigenvalues of **M**

$$\boxed{\det\left(\mathrm{M} - \lambda I\right) = \left(g^2 - \lambda\right)\left(g'^2 - \lambda\right) - g^2 g'^2 = 0}$$

with eigenvalues

$$\boxed{\lambda_1 = 0 \qquad \text{and} \qquad \lambda_2 = g^2 + g'^2}$$

In the diagonal basis **M** becomes

$$\boxed{\frac{v^2}{8}\left(A_\mu, Z_\mu\right)\begin{pmatrix} 0 & 0 \\ 0 & g^2 + g'^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \frac{1}{2}\left(A_\mu, Z_\mu\right)\begin{pmatrix} M_A^2 & 0 \\ 0 & M_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}}$$

where the A_μ and Z_μ fields correspond to eigenvectors of the mass matrix **M**.
The masses of the physical gauge bosons can be identified as

$$M_A = 0 \qquad\qquad M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

The physical sates, which correspond to the normalized eigenvectors of **M**, are

$$A_\mu = \left(g' W_\mu^3 + g B_\mu\right) \Big/ \sqrt{g^2 + g'^2} \qquad M_A = 0$$

$$Z_\mu = \left(g W_\mu^3 - g' B_\mu\right) \Big/ \sqrt{g^2 + g'^2} \qquad M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

By introducing θ_W (see L11)

$$g \sin \vartheta_W = e = g' \cos \vartheta_W$$

the fields A_μ and Z_μ can be re-expressed as

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \vartheta_W & \sin \vartheta_W \\ -\sin \vartheta_W & \cos \vartheta_W \end{pmatrix} \begin{pmatrix} B_\mu^0 \\ W_\mu^3 \end{pmatrix} \qquad \begin{aligned} A_\mu &= \cos \vartheta_W B_\mu^0 + \sin \vartheta_W W_\mu^3 \\ Z_\mu &= -\sin \vartheta_W B_\mu^0 + \cos \vartheta_W W_\mu^3 \end{aligned}$$

with $M_Z = \dfrac{1}{2} \dfrac{gv}{\cos \theta_W}$ and $M_W / M_Z = \cos \theta_W$

The GSW model is described by 4 free parameters, the $SU(2)_L \times U(1)_Y$ couplings g and g' (or e and θ_W), and the parameters μ and λ of the Higgs potential

$$v^2 = \frac{-\mu^2}{\lambda} \qquad \text{and} \qquad M_H = \sqrt{2\lambda} v$$

with v the vacuum expectation value of the Higgs field $v = \langle 0 | \phi | 0 \rangle = 246 \mathrm{GeV}$

The Glashow – Weinberg – Salam Model

In summary, the GWS model is based on the hypothesis that there exist two gauge fields. One of them (W^i, i = 1, 2, 3) has three components and corresponds to the adjoint representations of the gauge group $SU(2)_L$, while the second one (B) has one component and the gauge group is the group $U(1)_Y$.

The gauge group of the GWS model is the compact group $SU(2)_L \times U(1)_Y$ with couplings g and g'. Of the four particles, W^1 and W^2 are charged and W^3 and B are neutral.
As the fields W^3 and B have the same quantum numbers (Q=0, Y=0) mixing between them can take place. The physical neutral vector
particles, the photon and the Z boson are superpositions of the fields W^3 and B.

To make the vector bosons massive, one uses the mechanism of spontaneous symmetry breaking, for which one introduces and auxiliary two-component complex (4 d.o.f.) scalar field ϕ. Three of the four d.o.f. are used for providing via the Higgs mechanism, an additional polarization state for each of the three vector components, while the fourth one leads to the physical massive Higgs boson.

The vector boson sector of the GWS model is based on the Lagrangian L_{IVB}

$$\boxed{L_{IVB} = -\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \left| D_\mu \phi \right|^2 - \frac{\lambda}{4}\left(\phi^\dagger \phi - v^2\right)^2}$$

with the covariant derivative D_μ

$$\boxed{D_\mu \phi = \left(i\partial_\mu - igT_a \tau W^a_\mu - \frac{ig'}{2} YB_\mu \right)\phi}$$

Spontaneous symmetry breaking is realized when the second component of the field ϕ is shifted by a real constant v

$$\boxed{\phi(x) \to \phi(x) + \begin{pmatrix} 0 \\ v \end{pmatrix}}$$

As a result of the shift, the term $|D_\mu \phi|^2$ gives the following contribution to the mass matrix

$$\frac{g^2 v^2}{8}\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \frac{v^2}{8}\left(g W_\mu^3 - g' B_\mu\right)^2$$

After diagonalization of the mass matrix it becomes

$$\boxed{A_\mu = \left(g' W_\mu^3 + g B_\mu\right) \Big/ \sqrt{g^2 + g'^2} \qquad Z_\mu = \left(g W_\mu^3 - g' B_\mu\right) \Big/ \sqrt{g^2 + g'^2}}$$

The mass terms are given by

$$\boxed{M_A = 0 \qquad M_W = \frac{v}{2} g \qquad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2} = M_W / \cos\theta_W}$$

The last term in L_{IVB} can be rewritten as

$$-\frac{\lambda}{16}\left(\Phi^2 + h^2\right)^2 - \frac{\lambda v}{4} h\left(\Phi^2 + h^2\right) - \frac{\lambda v^2}{2\sqrt{2}} h^2 \qquad \boxed{v = \langle 0|\phi|0\rangle = 246\,\mathrm{GeV}}$$

with $\Phi^2 = \phi_1^2 + \phi_2^2 + \phi_4^2$, from which it follows that the filed h has mass $M_h = \sqrt{\lambda} v$, while the components ϕ_i represent Goldstone fields. They can be removed by a gauge transformation, as a result of which three components of the non-Abelian gauge field will acquire a third polarization component (the Higgs mechanism).

Fermion Masses

The mass term

$$\boxed{-m\bar{\psi}\psi = -m\bar{\psi}\left[\frac{1}{2}\left(1-\gamma^5\right)+\frac{1}{2}\left(1+\gamma^5\right)\right] = -m\left(\bar{\psi}_R\psi_L+\bar{\psi}_L\psi_R\right)}$$

does not respect the $SU(2)_L \times U(1)_Y$ gauge symmetry because ψ_L is an $SU(2)_L$ doublet and ψ_R a singlet with different transformation properties.

Therefore, a term which mixes left and right handed particles cannot be gauge invariant.

Remarkably the Higgs mechanism can also be used to give masses to fermions, without the introduction of new particles.

The same $SU(2)_L$ local gauge transformation of the Higgs field

$$\boxed{\phi \rightarrow \phi' = \left(I + ig\vec{\alpha}(x)\cdot\vec{T}\right)\phi}$$

applies also to ψ_L

$$\boxed{\begin{aligned} \psi_L \rightarrow \psi_L' &= \left(I + ig\vec{\alpha}(x)\cdot\vec{T}\right)\psi_L \\ \bar{\psi}_L \equiv \psi_L^{\dagger}\gamma^0 \rightarrow \bar{\psi}_L' &= \bar{\psi}_L\left(I - ig\vec{\alpha}(x)\cdot\vec{T}\right) \end{aligned}}$$

It follows that $\bar{\psi}_L\phi$ is invariant under $SU(2)_L$ transformations and $\bar{\psi}_L\phi\psi_R$ under $SU(2)_L \times U(1)_Y$ transformations.

Therefore the term $\boxed{-g_f\left(\bar{\psi}_L\phi\psi_R+\bar{\psi}_R\phi^{\dagger}\psi_L\right)}$ is invariant under $SU(2)_L \times U(1)_Y$ transformations, note $\left(\bar{\psi}_L\phi\psi_R\right)^{\dagger} = \left(\bar{\psi}_R\phi^{\dagger}\psi_L\right)$

To generate the mass term for the charged leptons we couple the Higgs field to the leptons doublet as

$$\boxed{L_e = -g_e \left[\left(\bar{\nu}_e, \bar{e} \right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R \left(\phi^{+*}, \phi^{0*} \right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]}$$

g_e – Yukawa coupling constant of the electron to the Higgs field, which is not predicted by the SM and which must be determined from the experiment.

Yukawa coupling ≡ coupling of a fermion field with a scalar field (i.e. NπN)

After spontaneous symmetry breaking of the Higgs doublet,

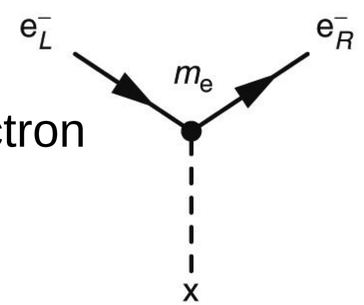
with the substitution $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ the Lagrangian becomes

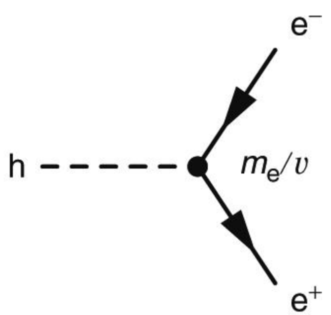
$$\boxed{L_e = -\frac{g_e}{\sqrt{2}} v \left(\bar{e}_L e_R + \bar{e}_R e_L \right) - \frac{g_e}{\sqrt{2}} h \left(\bar{e}_L e_R + \bar{e}_R e_L \right) = -m_e \bar{e} e - \frac{m_e}{v} \bar{e} e h}$$

where the Yukawa coupling g_e
has been identified with the electron mass m_e.
Note that the neutrino remains massless.

$$\boxed{g_e = \sqrt{2} m_e / v \qquad m_e = \frac{1}{\sqrt{2}} g_e v}$$

first term in L_e :
coupling of the electron
to the Higgs field
gives mass to e





second term in L_e :
coupling between the electron
and the Higgs boson
(interaction term)

In addition to give mass to the electron (fermion) via the coupling of the left and right handed components to the Higgs field, the Higgs mechanism introduces also a direct interact between the fermion and the Higgs boson itself.
Note that the coupling to the Higgs field is flavor conserving.

So far, only the lower components of the weak isospin doublet are present!
This mechanism can be used to generate the masses of the down quarks but not of the up quarks.

To give masses to up quarks consider the conjugate doublet ϕ_C
(recall the isospin transformation for antiparticles)

$$\boxed{\phi_C = -i\sigma_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}}$$

which transforms exactly in the same way as ϕ (property of SU(2) only).

To generate the mass term for the up quarks we couple the Higgs field as

$$\boxed{L_u = -g_u\left[\left(\bar{u},\bar{d}\right)_L\begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}u_R + \bar{e}_R\left(-\phi^0,\phi^{-*}\right)\begin{pmatrix} u \\ d \end{pmatrix}_L\right]}$$

which, after spontaneous symmetry breaking, becomes

$$\boxed{L_u = -\frac{g_u}{\sqrt{2}}v\left(\bar{u}_L u_R + \bar{u}_R u_L\right) - \frac{g_u}{\sqrt{2}}h\left(\bar{u}_L u_R + \bar{u}_R u_L\right) \rightarrow L_u = -m_u\bar{u}u - \frac{m_u}{v}\bar{u}uh}$$

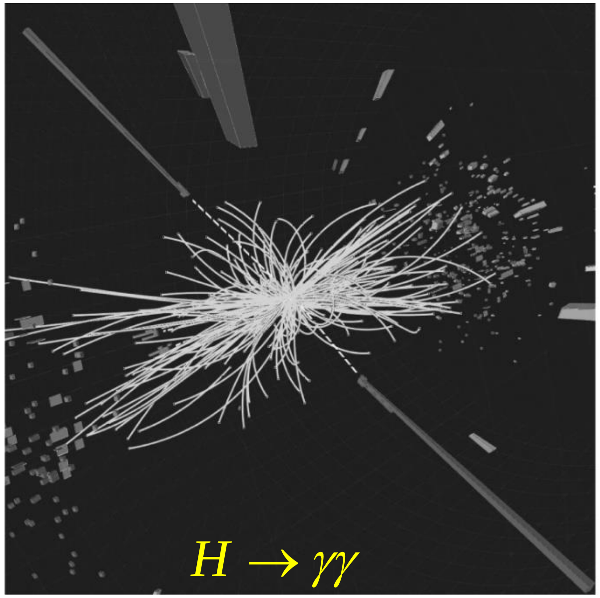
Finally, the gauge invariant mass terms for all fermions can be constructed from

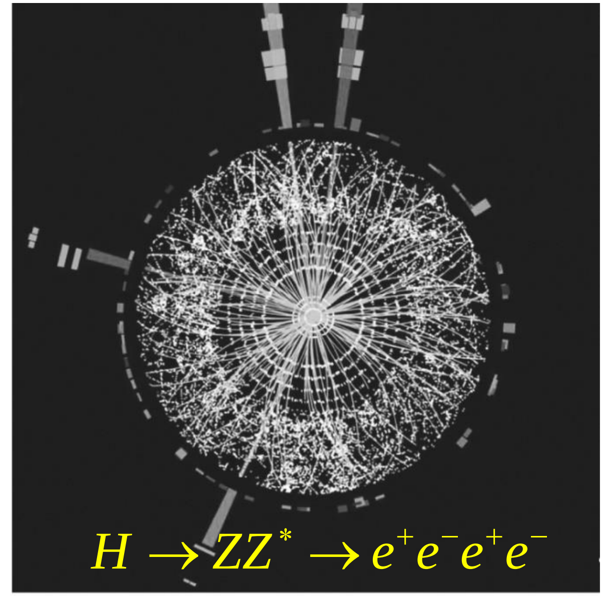
$$\boxed{L = -g_f\left(\bar{\psi}_L \phi \psi_R + \bar{\psi}_R \phi^\dagger \psi_L\right) \qquad \text{or} \qquad L = g_f\left(\bar{\psi}_L \phi_C \psi_R + \bar{\psi}_R \phi_C^\dagger \psi_L\right)}$$

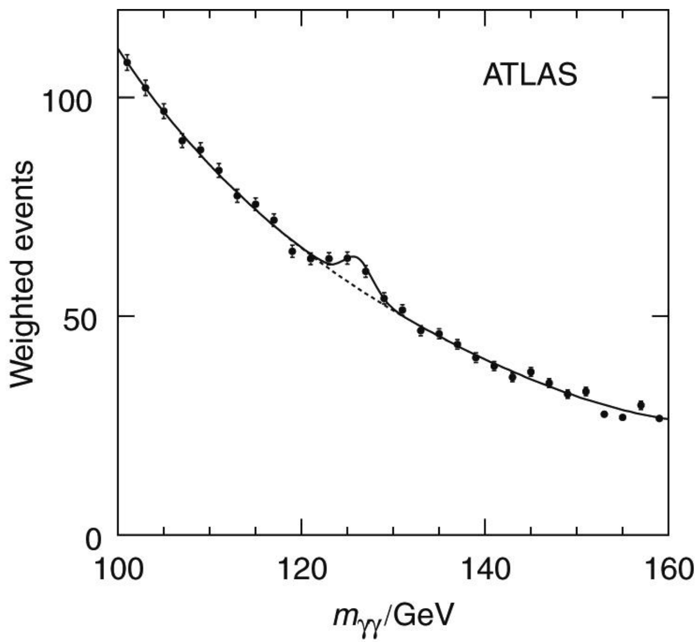
with the Yukawa couplings of the fermions to the Higgs field

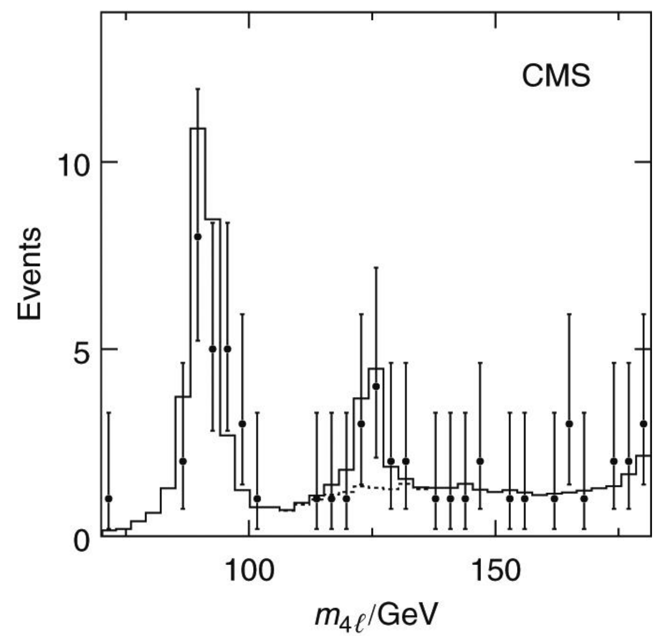
$$\boxed{g_f = \sqrt{2} m_f / v \qquad \rightarrow \qquad m_f = \frac{g_f v}{\sqrt{2}}}$$

The Discovery of the Higgs Boson (2012)









Properties of the Higgs Boson

Neutral spin-0 particle with weak isospin T = 1/2, T_3 = -1/2, and weak hypercharge Y = 1

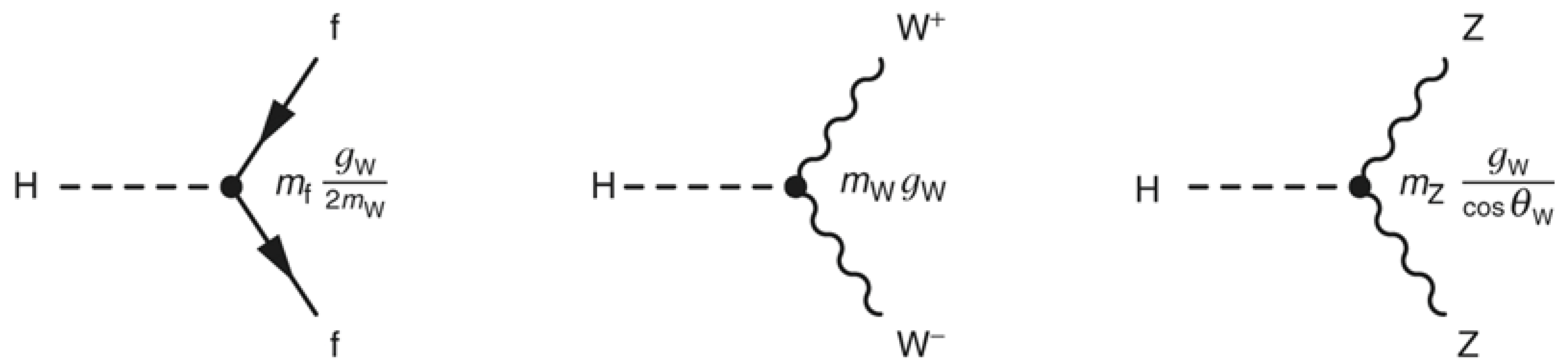
The Higgs mass is a free parameter of the SM $m_H = \sqrt{2\lambda} v \approx 125.10 \pm 0.14\ \text{GeV}$

$$\Gamma_H < 0.013\ \text{GeV}$$

The Higgs boson couples to all fermions with a coupling strength proportional to the fermion mass $\boxed{g_f = \sqrt{2} m_f / v}$

Feynman rules for the Yukawa interaction vertex

$$\boxed{-i\frac{m_f}{v} = -i\frac{m_f}{2M_w} g}$$



The resulting matrix element is proportional to the mass of the particle, which couples to the Higgs boson

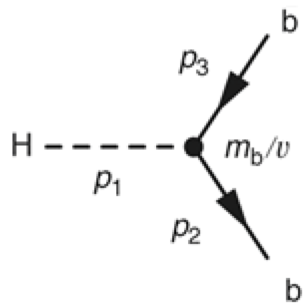
$\Rightarrow$ determines dominant process for production and decay of the Higgs boson.

Higgs Boson Decay

The Higgs boson can decay to all SM particles provided that $m_f < 1/2\ m_H$.
Because of the coupling, the Higgs boson decay predominantly to heaviest particles which are energetically allowed (largest BR).

H – scalar particle $\rightarrow$ no polarization

$H \rightarrow b\bar{b}$



$$M_{fi} = \bar{u}(p_2)\, m_b \frac{g}{2M_W} v(p_3) = \frac{m_b}{v} u^\dagger \gamma^0 v = \frac{m_b}{v} 2E = \frac{m_b}{v} m_H$$

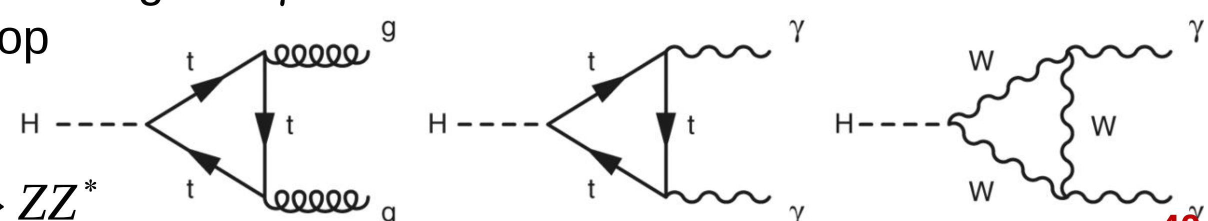
$$\langle |M|^2 \rangle = |M_{\uparrow\uparrow}|^2 + |M_{\downarrow\downarrow}|^2 = \frac{m_b^2}{v^2} 8E^2 = \frac{2m_b^2}{v^2} m_H^2 = \frac{g^2}{2M_W^2} m_b^2 m_H^2$$

and using Fermi's golden rule

$$\boxed{\Gamma\left(H \rightarrow b\bar{b}\right) = 3 \times \frac{g^2}{32\pi M_W^2} m_b^2 m_H \sim O(2\mathrm{MeV})}$$

Remember that also the masses run with q^2 (not only α_S),
hence the masses appearing in Γ are estimated at $q^2 = m_H^2$ (m_b = 3 GeV).

The Higgs boson can decay also to 2 g or 2 γ
via a virtual top quark or a W loop



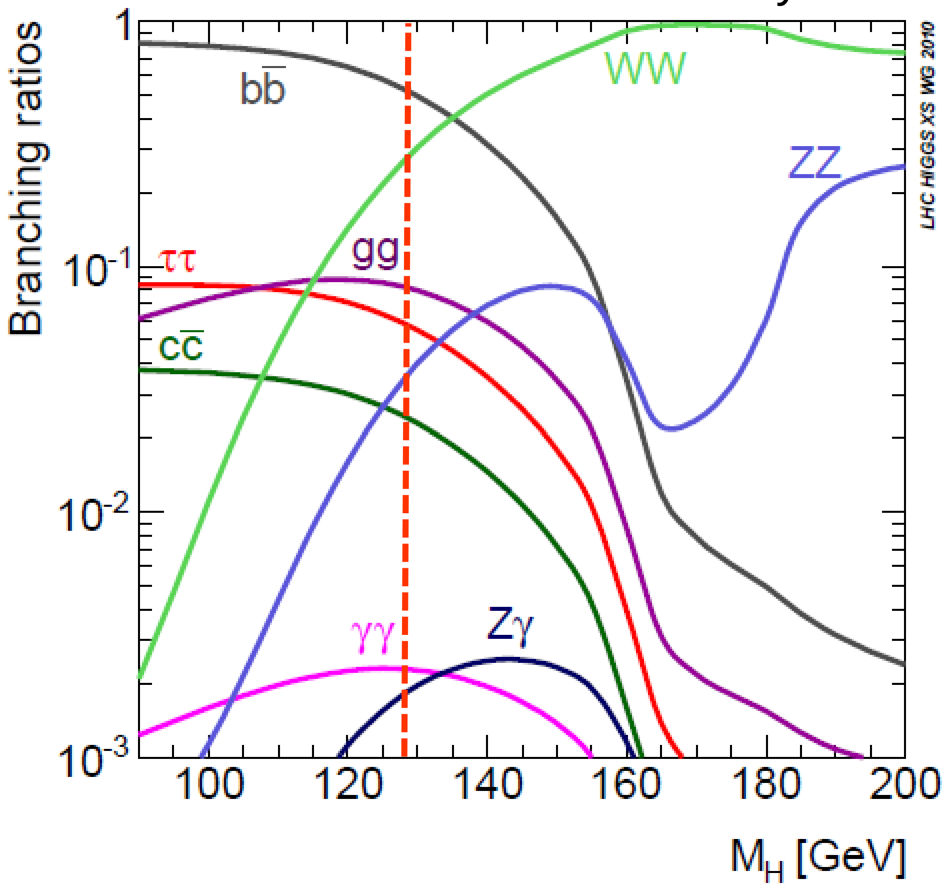
or $H \rightarrow WW^*$ and $H \rightarrow ZZ^*$

Higgs Decay Branching Ratios

Higgs boson couplings are proportional to the mass

$\Rightarrow$ Higgs decays predominantly to heaviest particles which are energetically allowed

calculated before discovery



$$\boxed{m_H \approx 125.10 \pm 0.14 \text{ GeV}}$$

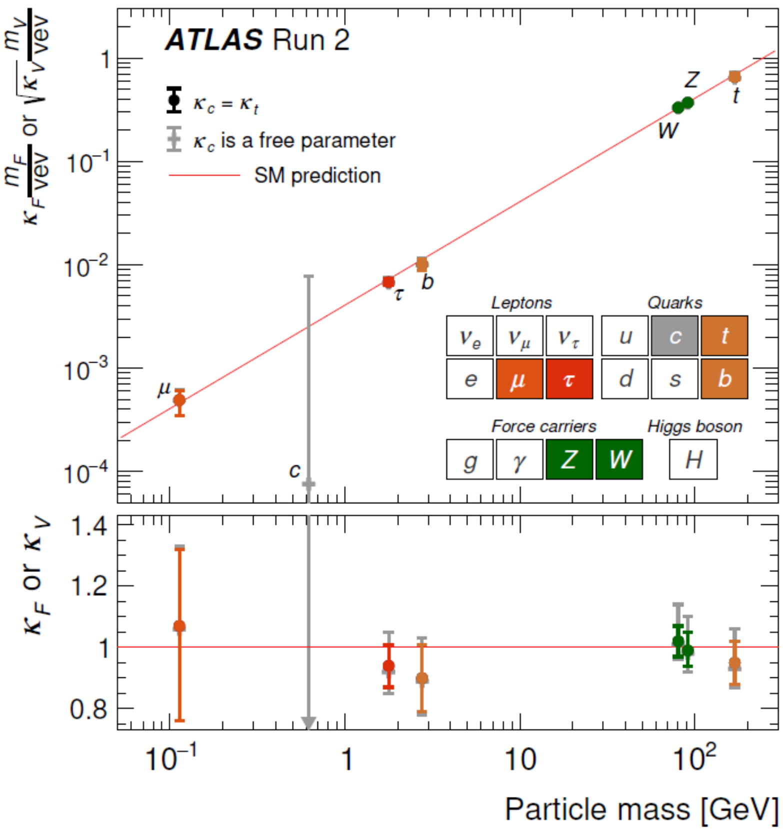
Higgs decay branching ratios at 125 GeV (calculated)

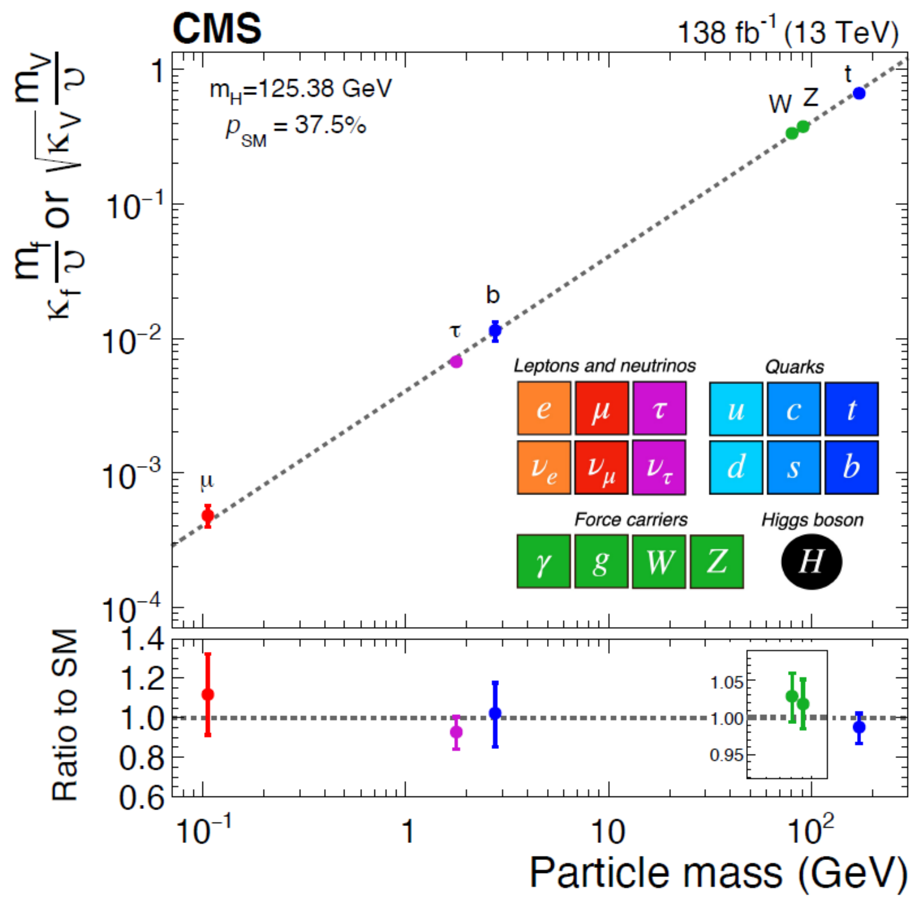
$H \to b\bar{b}$	57.8%
$H \to W^+W^-$	21.6%
$H \to \tau^+\tau^-$	6.4%
$H \to gg$	8.6%
$H \to c\bar{c}$	2.9%
$H \to ZZ^*$	2.7%
$H \to \gamma\gamma$	0.2%

B.R. measurements are under way.

Higgs Couplings

$$g_f = \sqrt{2} m_f / v$$





in excellent agreement with data

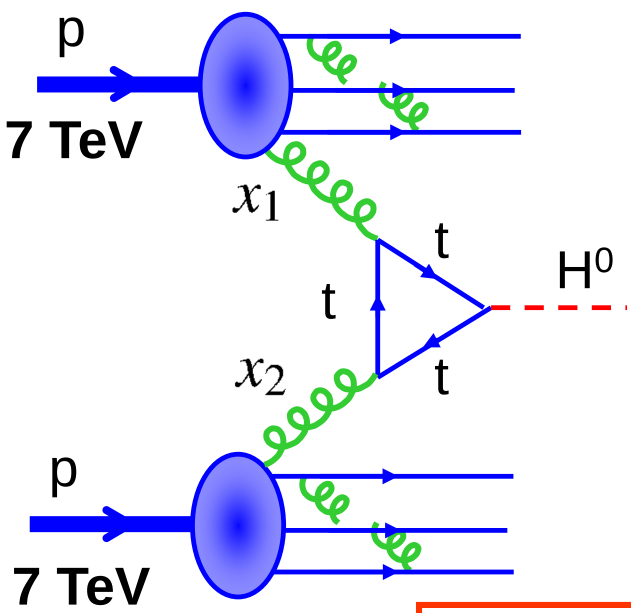
Higgs Production in Hadronic Collisions

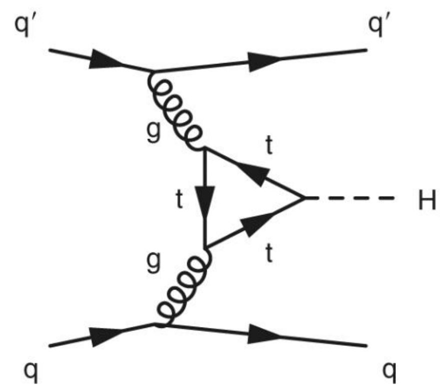
Higgs production at the Large Hadron Collider LHC

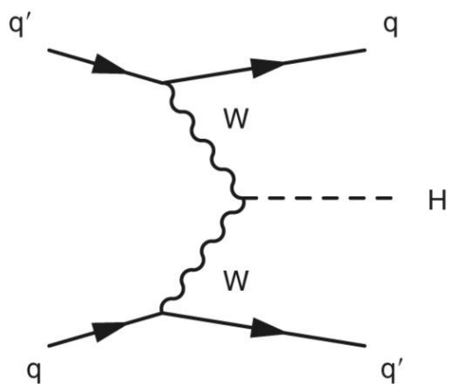
The LHC will collide 7 TeV protons on 7 TeV protons

However underlying collisions are between partons

Higgs production the LHC dominated by "gluon-gluon fusion"







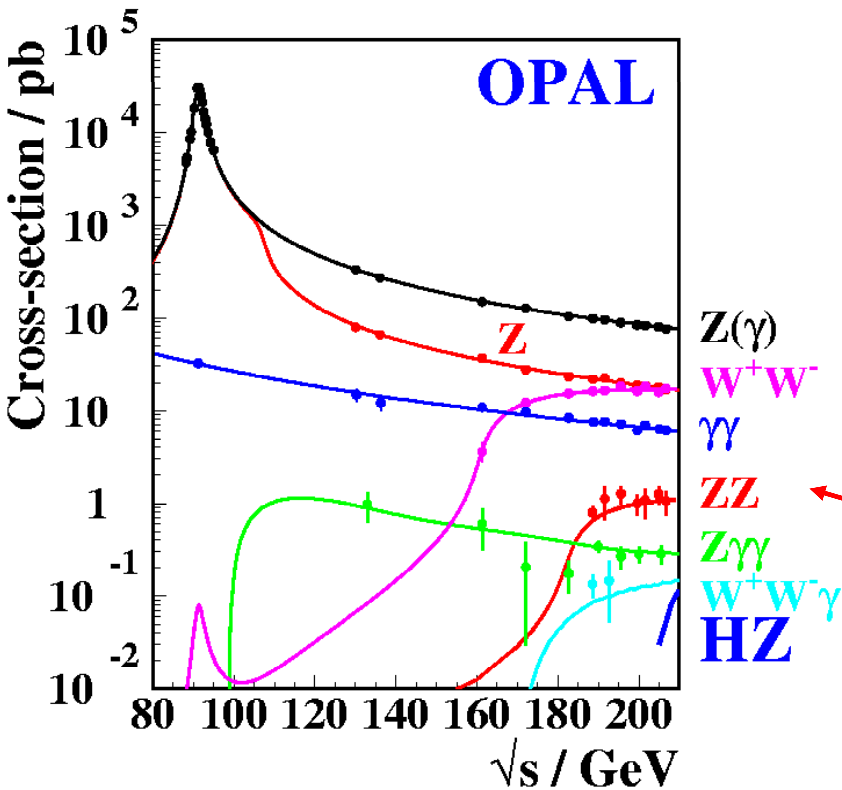
$$\boxed{\sigma\left(pp \to HX\right) \sim \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, g(x_1)\, g(x_2)\, \sigma\left(gg \to H\right)}$$

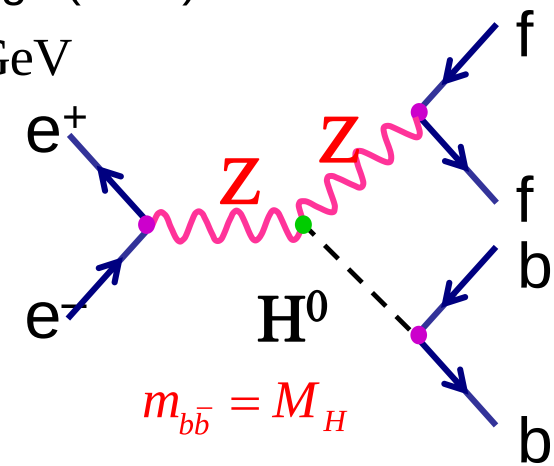
Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section.

Higgs Boson Production in e⁺e⁻ Annihilations

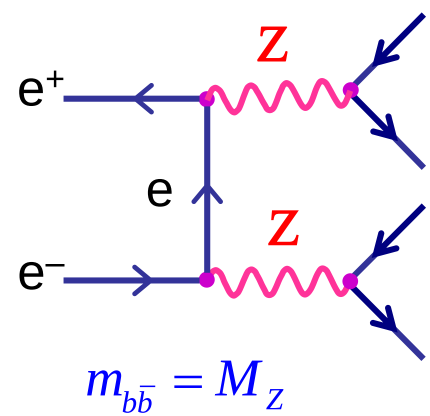
Clear experimental signature ("Higgsstrahlung"),
but small cross section with large "backgrounds",
Need enough energy to produce a Z boson and the Higgs (ILC?):

$$2E_{\text{beam}} = \sqrt{s} > M_Z + M_H = 217\ \text{GeV} \quad \Rightarrow \quad E_{\text{beam}} > 115\ \text{GeV}$$





main background to "Higgssthralung"



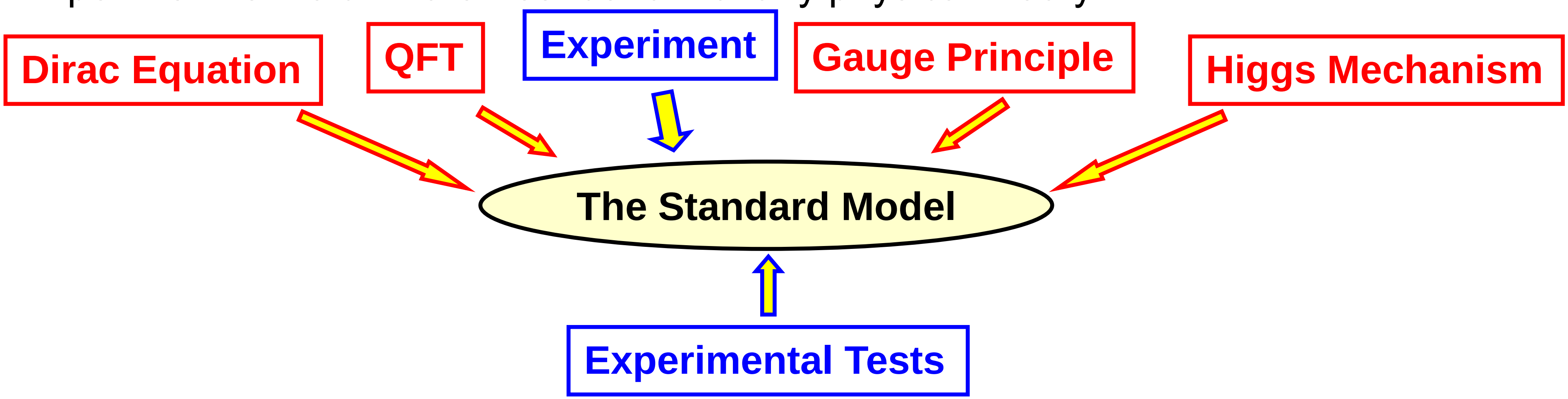
The only way to distinguish the two diagrams is from the invariant mass of the jets from the boson decays.

Concluding Remarks

We covered almost all aspects of modern particle physics.
We did not discuss BSM physics, i.e. ν masses, dark matter, (anti)matter asymmetry, …

The Standard Model, now completed with the discovery of the Higgs boson, is one of the greatest scientific triumphs of the late 20th century.
Many people contributed to this success with their insight and hard work.

The Standard Model developed through close interplay of experiment and theory.
Experiment is the ultimate "test bench" of any physical theory.



Experimental particle physics provides many precise measurements.
The Standard Model describes (almost) all current data.

Despite its great success, do not forget that it is just a model: a collection of beautiful theoretical ideas put together to fit with experimental data.
There remains many issues / open questions / new discoveries / …

Standard Model Open Issues

The Standard Model has too many free parameters (19 + 7 = 26):

$$m_e, m_\mu, m_\tau; \quad m_u, m_d, m_c, m_s, m_t, m_b \qquad m_{\nu_1}, m_{\nu_2}, m_{\nu_3};$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}^{q}; \quad e, g_W, \alpha_S; \;\; v, M_H; \;\; \theta_{CP} \qquad \theta_{12}^{\nu}, \theta_{13}^{\nu}, \theta_{23}^{\nu}, \delta_{CP}^{\nu};$$

Why three generations?

Why $SU(3)_C \times SU(2)_L \times U(1)_Y$?

Unification of Forces

Origin of CP violation in early Universe and of the matter – antimatter asymmetry

What is the Dark Matter? and the Dark Energy?

Why is the weak interaction V – A?

Massive neutrinos

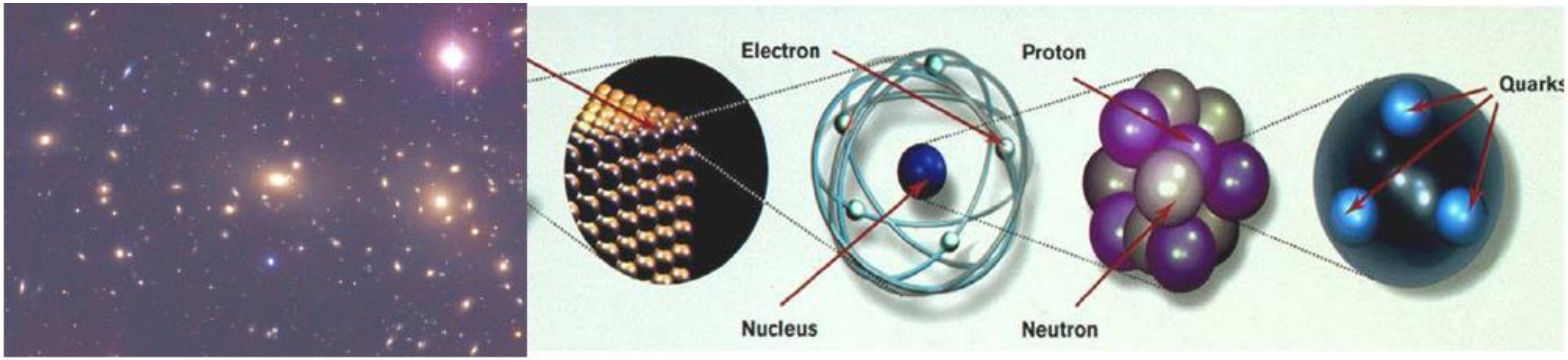
Why are neutrinos so light?

Where are the right-handed neutrinos?

The Higgs field gives rise to a huge cosmological constant

Ultimately need to include gravity

The Dark Mystery of Matter



What stuff is the Universe made of ??

- **Elementary Particles** 0.1%
  - ⇨ 12 **matter particles** (quarks, leptons)
    - only 4 relevant today (u, d, e, ν)
  - ⇨ 13 **force particles** (3 massive, 10 massless)
- **Composite Particles (hadrons)** 5%
  - ⇨ hundreds…
    - only 2 are relevant (p,n), making nuclei
- **Dark Matter** 25%
  - ⇨ made of **unknown particles**
- **Dark Energy** 70%
  - ⇨ **vacuum energy**
    - of completely unknown origin
  - ⇨ should be infinite or exactly 0

Higgs mechanism

QCD chiral symmetry breaking

We don't know **how** and **why** for ~ 5%

We don't even know **what** for the other 95%