cosmic rays and gamma rays

→ *Galactic cosmic rays*
• energetics of the sources
• supernova remnants: photons observed?

→ *extragalactic cosmic rays*
• energetics of the sources
• gamma ray bursts
• active galaxies

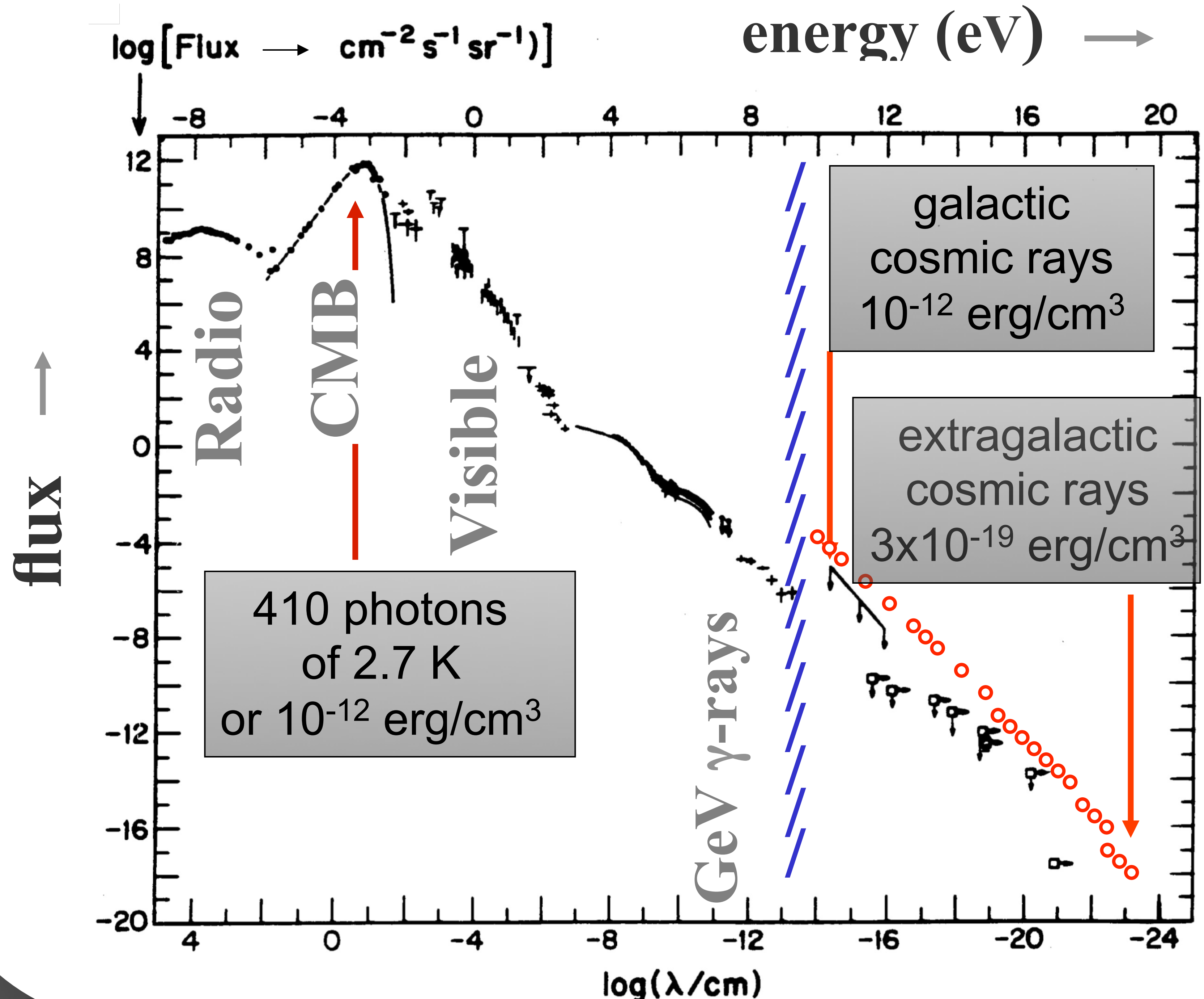
→ *general*
• shock acceleration
• an example: supernova remnants

total flux = velocity x density

$$\boxed{4\pi\int dE\,(\,E\,\frac{dN}{dE}\,) = c\,\rho_E}$$

energy density is the key !

$1\;TeV = 1.6\;erg$



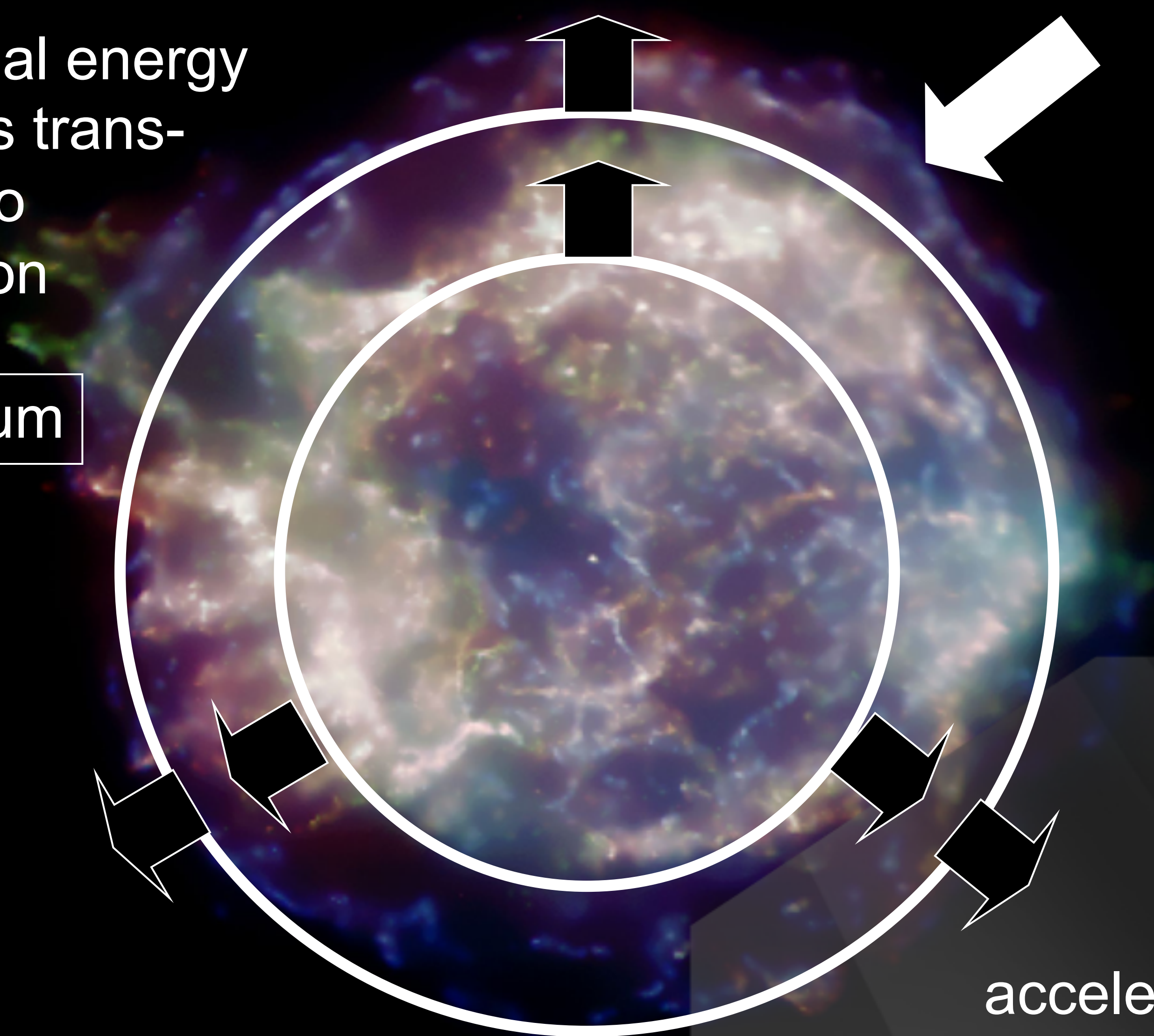
energy (eV)
log[Flux → cm^-2 s^-1 sr^-1)]
-8
-4
0
4
8
12
16
20
12
8
4
0
-4
-8
-12
-16
-20
flux
Radio
CMB
Visible
GeV γ-rays
galactic
cosmic rays
10^-12 erg/cm^3
extragalactic
cosmic rays
3x10^-19 erg/cm^3
410 photons
of 2.7 K
or 10^-12 erg/cm^3
4
0
-4
-8
-12
-16
-20
-24
log(λ/cm)

cassiopeia A supernova remnant in X-rays

gravitational energy released is trans-formed into acceleration

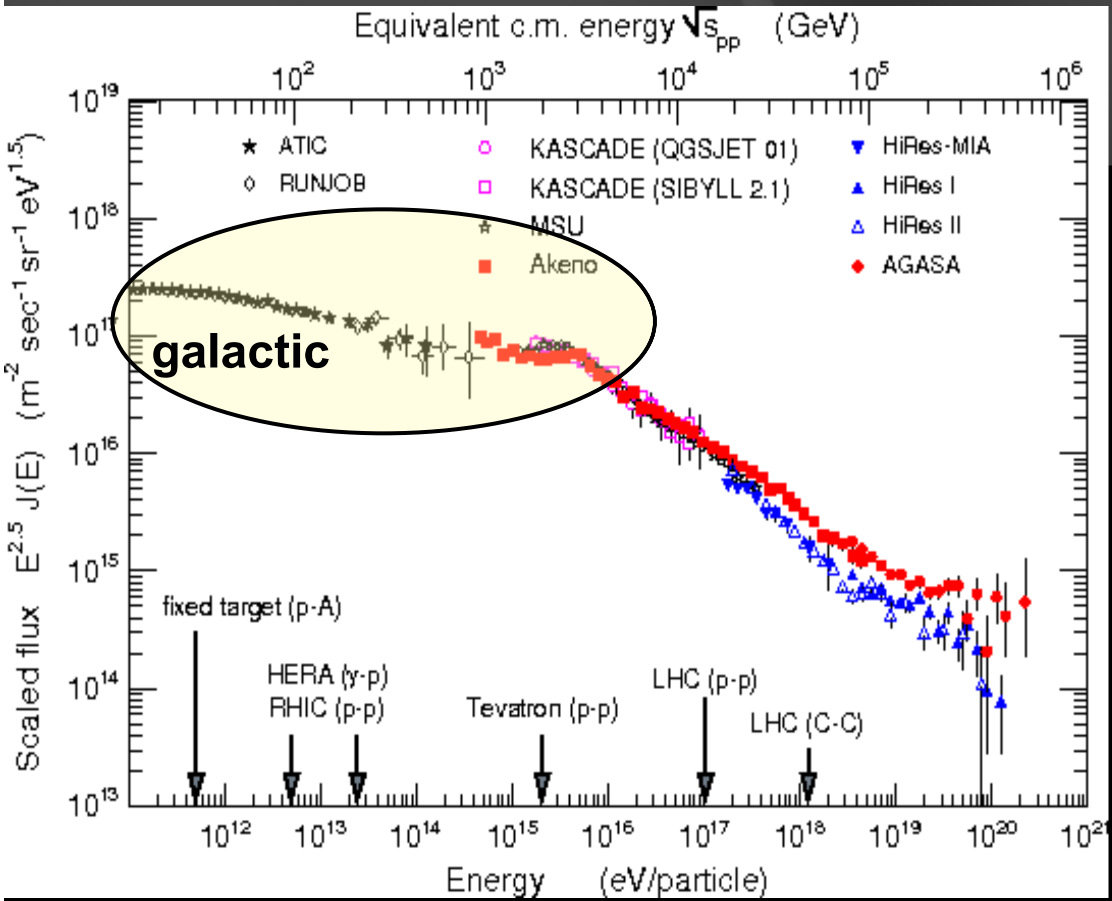
→

E^{-2} spectrum



acceleration when particles cross high B-fields

Cosmic Rays & SNRs



observed energy density of galactic CR:

$\sim 10^{-12}$ erg/cm^3

supernova remnants: 10^{50} ergs every 30 years

$\sim 10^{-12}$ erg/cm^3

for steady state of CR with lifetime 10^6 years

SNRs provide the environment and energy to explain the galactic cosmic rays!

Steady state cosmic ray flux in the Galaxy

$\rightarrow \rho_{cr} = 10^{-12}\, erg\ cm^{-3}\ or\ 10^{-26}\, erg\ cm^{-3} s^{-1}\ for$
$particles\ with\ an\ average\ confinement$
$time\ in\ the\ galaxy\ of\ 3 \times 10^{6}\ years$

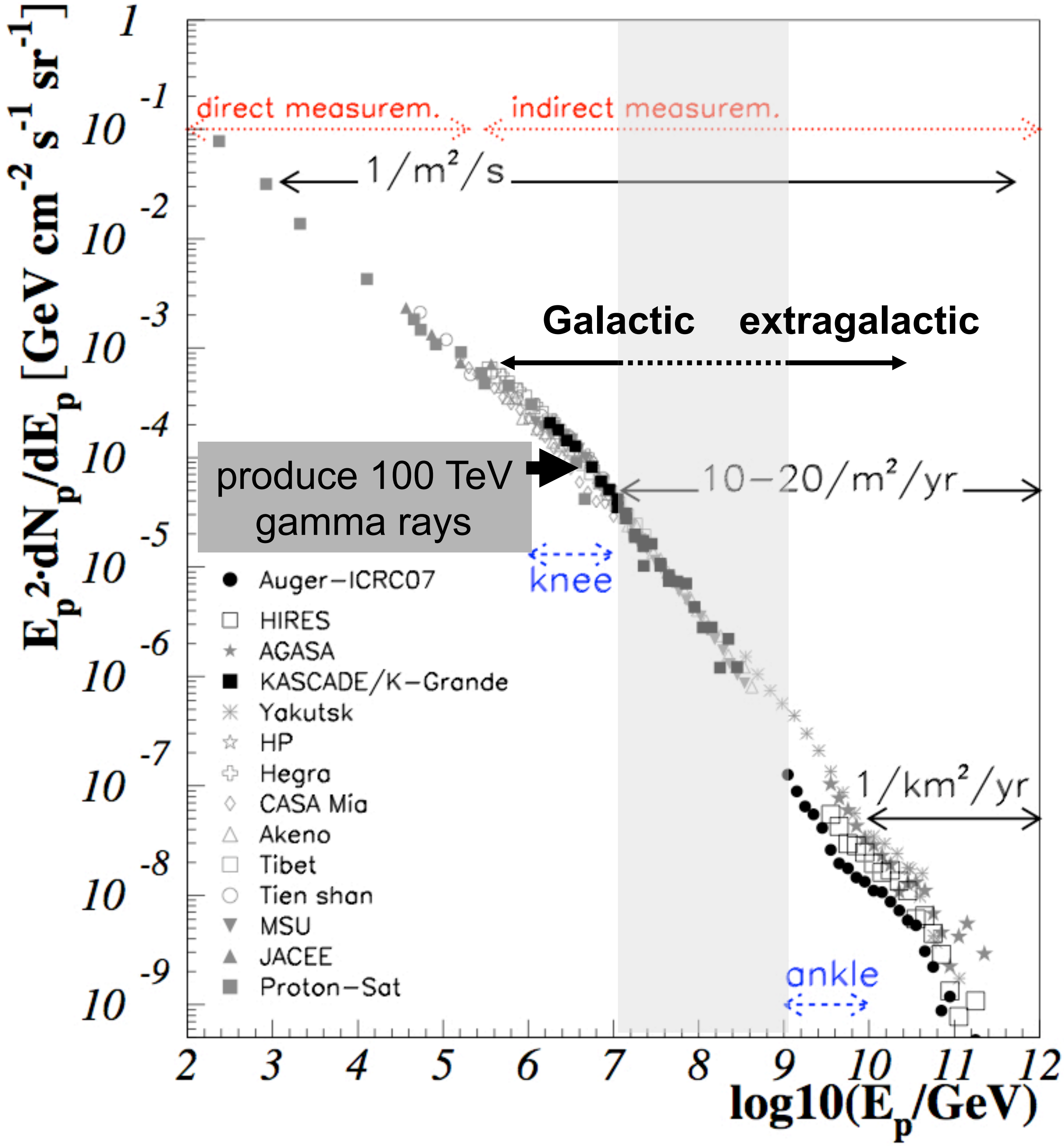
$\rightarrow this\ requires\ accelerators\ delivering\ 10^{41}\, ergs^{-1}$
$in\ a\ volume\ of\ 10^{67}\, cm^{3}$

$\rightarrow W = 10^{50}\, erg\ every\ 30\ years\ is\ 10^{41}\, ergs^{-1}$

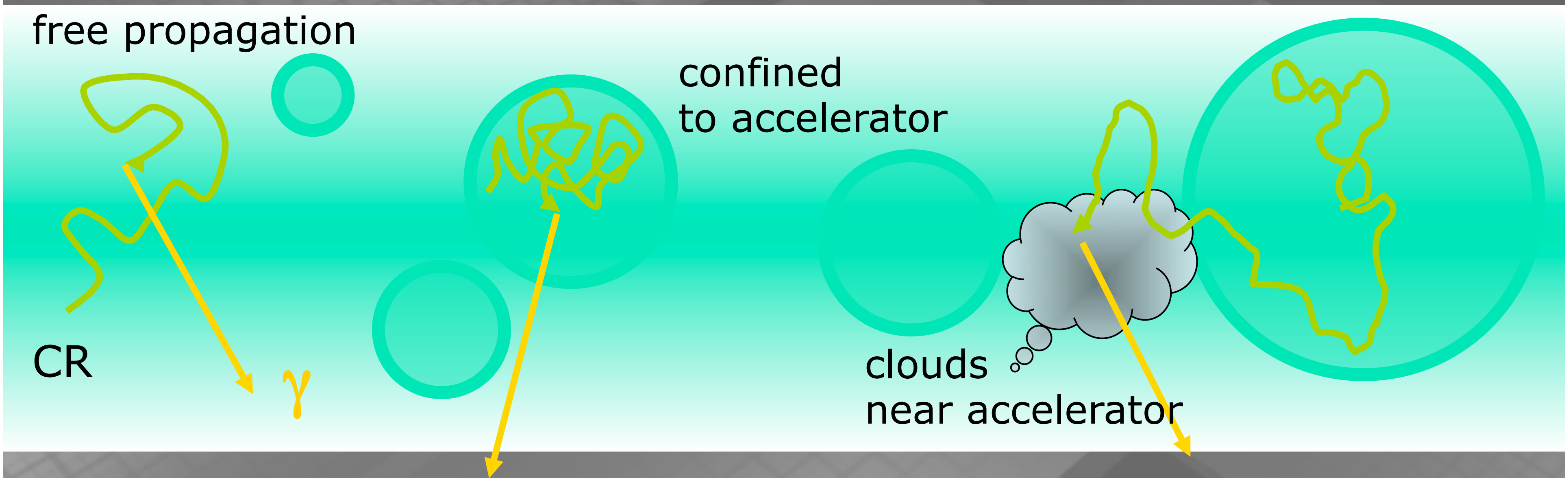
1 solar mass (10^{53} erg) x 0.01 (lost to ν's) x 0.1 efficiency

Galactic cosmic rays:

PeVatrons ?



origin of galactic cosmic rays

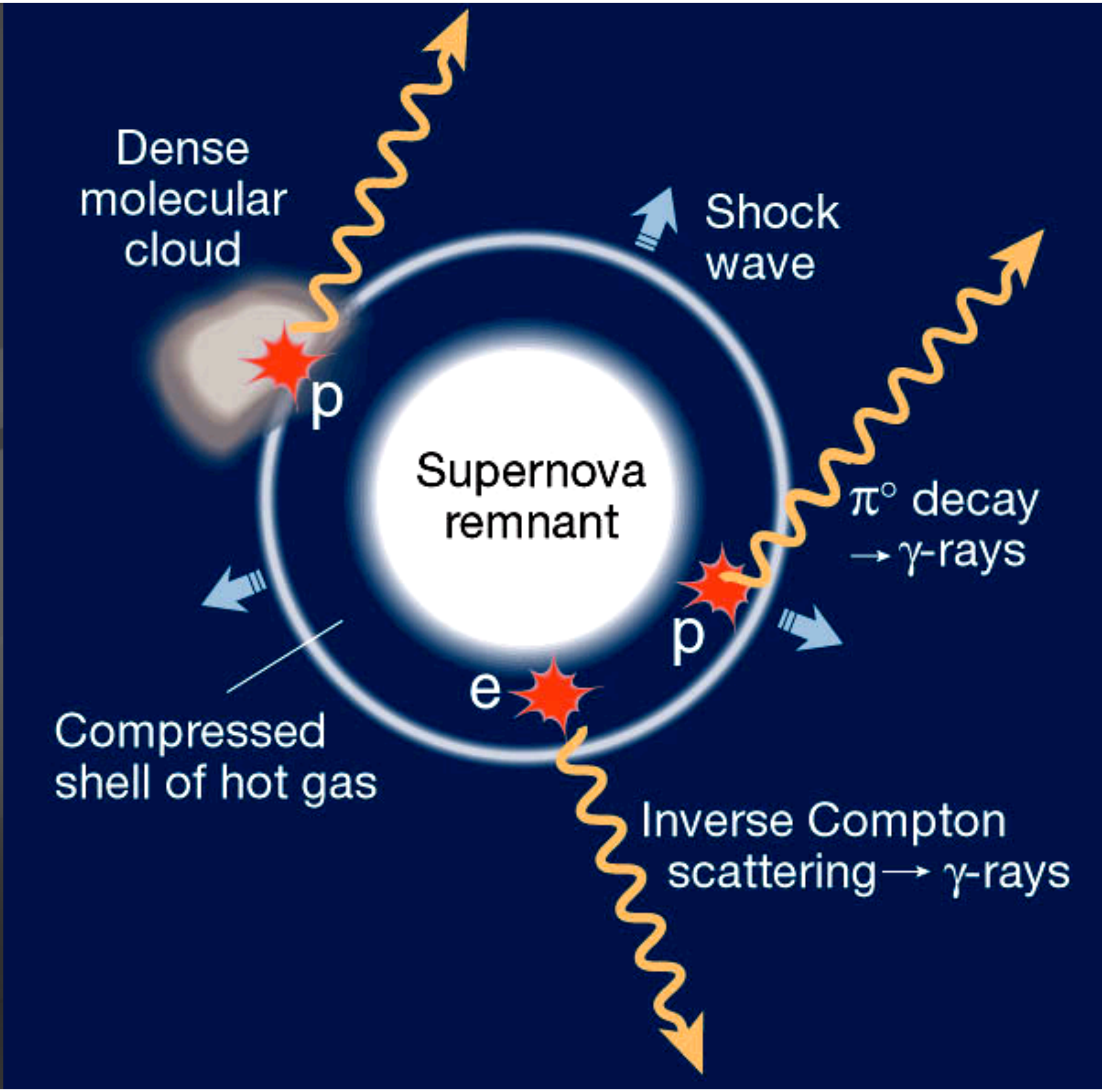


VHE gamma rays from secondary interactions:

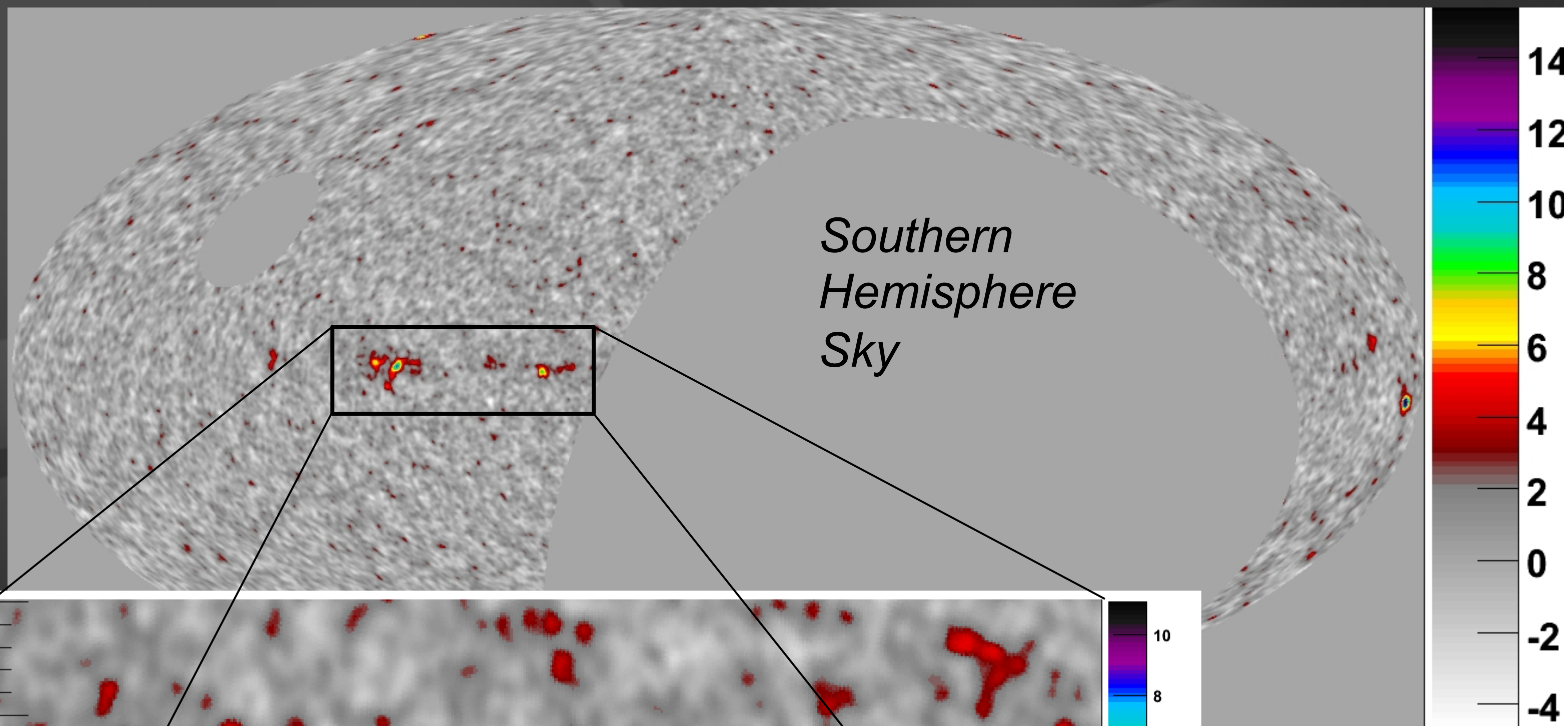
- p: π^o production and decay
- e: Inverse Compton scattering and bremsstrahlung
- trace beam density x target density

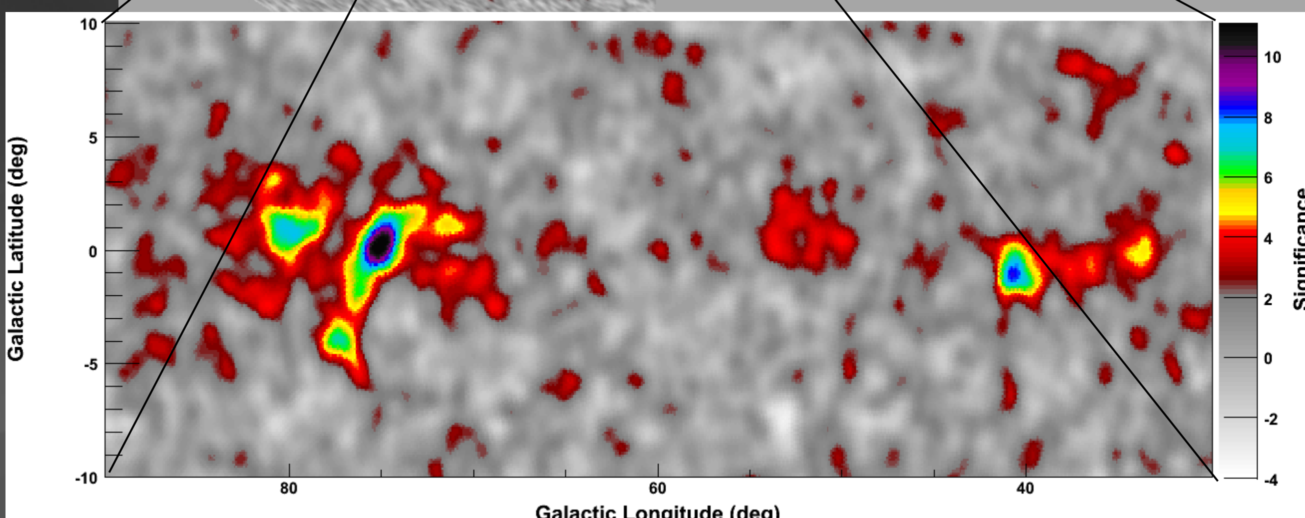
neutrinos from supernova remnants :

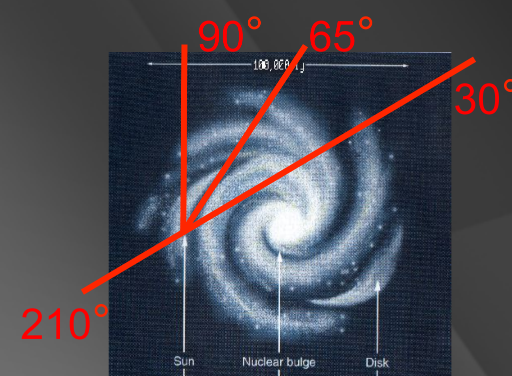
molecular clouds in star-forming regions where super-nova explode: beam dumps!



galactic plane in 10 TeV gamma rays :
supernova remnants in star forming regions







milagro

emissivity (units: (note!) per unit volume per GeV per second) in photons produced by a number density of cosmic rays N_p interacting with a target density n_{gas} per cm^3

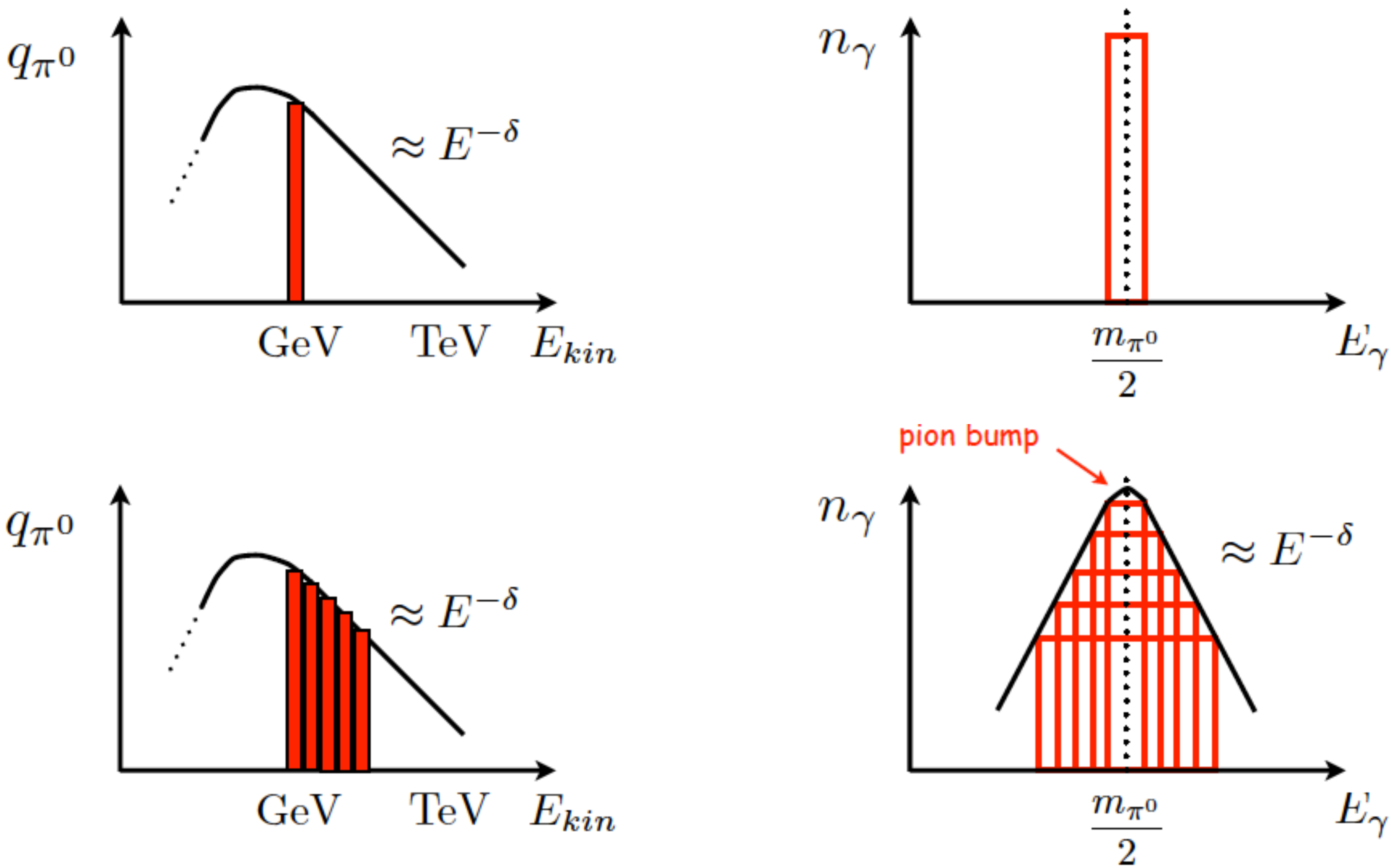
production rate

total cross section

$$q_{\pi^0} = \int \mathrm{d}E_p \; N_p(E_p) \; \delta(E_{\pi^0} - f_{\pi^0} E_{p,kin}) \; \sigma_{pp}(E_p) \; n_{gas} \; c$$

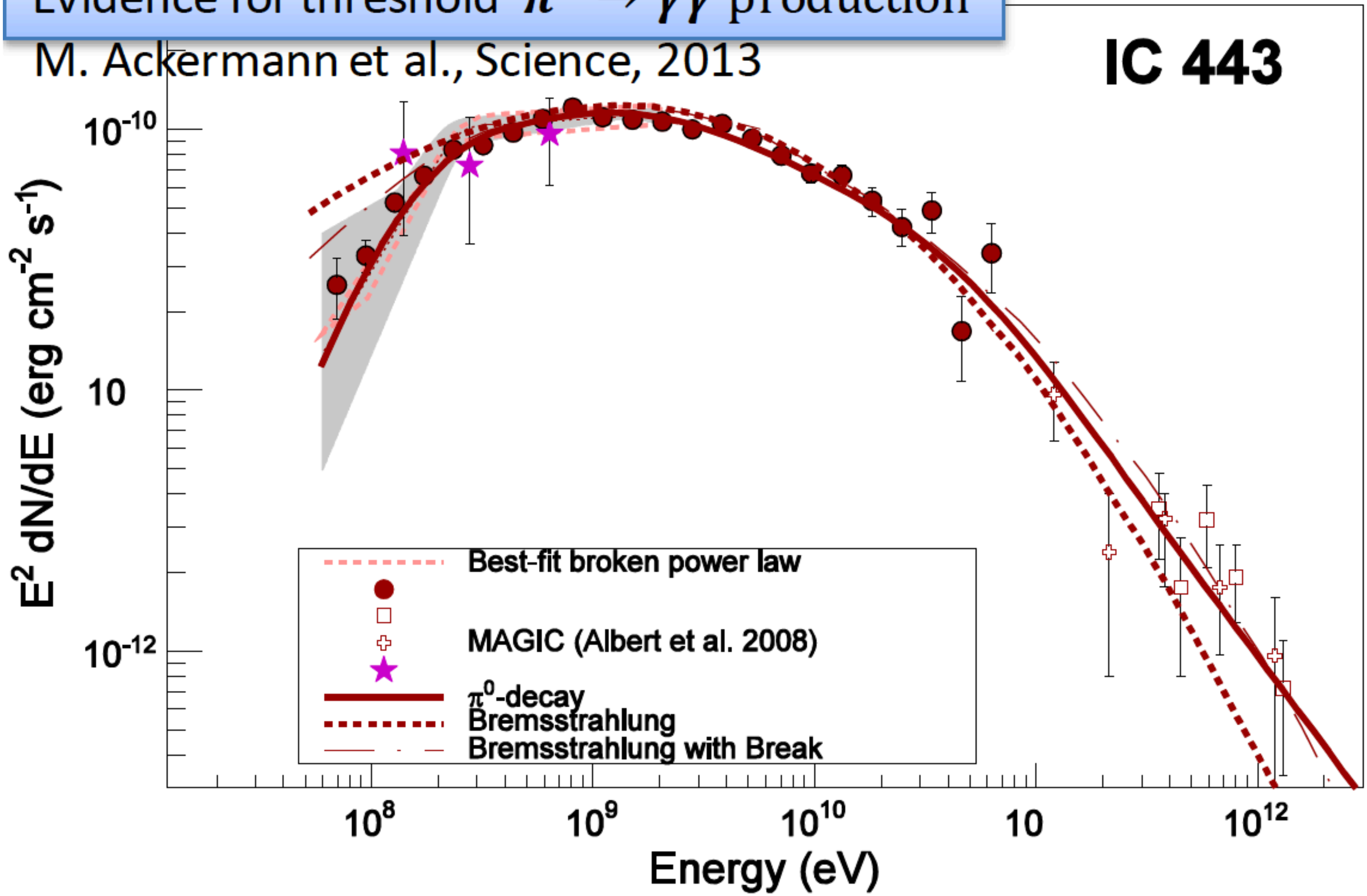
$$\mathrm{q}_\gamma\left(E_\gamma\right) = 2q_\pi\left(\frac{E_\pi}{2}\right)$$

pi-zero bump at $m_\pi/2$



Evidence for threshold $\pi^0 \rightarrow \gamma\gamma$ production

M. Ackermann et al., Science, 2013



$$\int_{1\mathrm{TeV}} dE_\gamma \, E_\gamma \, \frac{dN_\gamma}{dE_\gamma} = \frac{1}{4\pi \, d^2} L_\gamma$$

$$L_\gamma = V \, Q_\gamma = \frac{W}{\rho_{cr}} Q_\gamma$$

volume of the remnant

10^{-12} erg/cm^3

energy in >TeV photons produced by cosmic rays per cm^3 per sec

γ, ν flux of galactic cosmic rays

a SNR at d = 1 kpc transferring W = 10^{50} erg to cosmic rays interacting with interstellar gas (or molecular clouds) with density n > 1 cm^{-3} produces a gamma-ray flux of

$$E\frac{dN_\gamma}{dE}(>1\,TeV) =$$

$$\geq 10^{-11} cm^{-2} s^{-1} \frac{W}{10^{50}\,erg} \frac{n}{1\,cm^3} \left(\frac{d}{1\,kpc}\right)^{-2}$$

should be observed by present TeV gamma-ray telescopes

Milagro sources ?
RX J1713.7-3946??

neutral pions

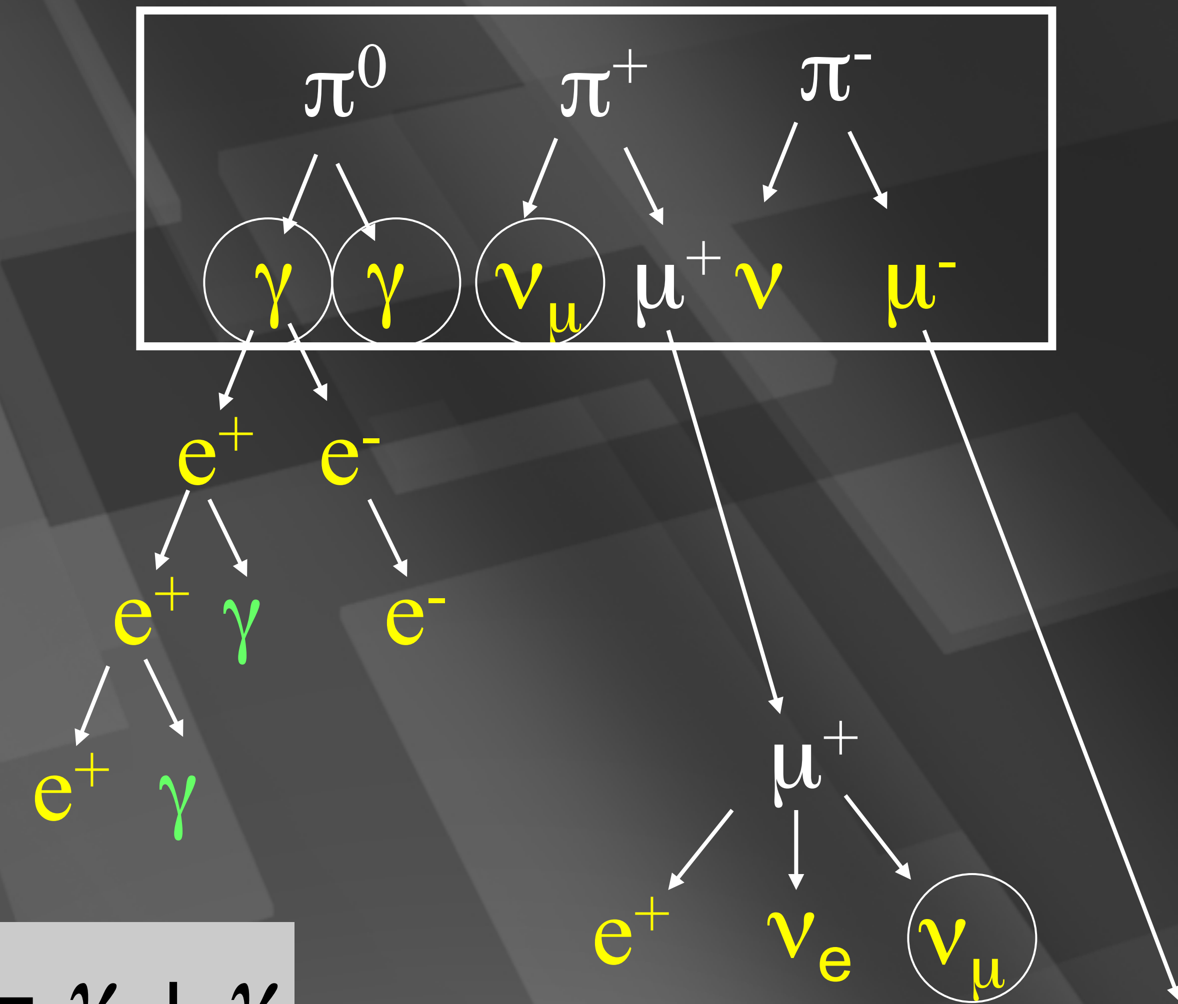
are observed as

gamma rays

charged pions

are observed as

neutrinos



$$\nu_\mu + \overline{\nu}_\mu = \gamma + \gamma$$

ν flux accompanying TeV gammas

$$\frac{dN_\nu}{dE} \cong \frac{1}{2}\frac{dN_\gamma}{dE}$$

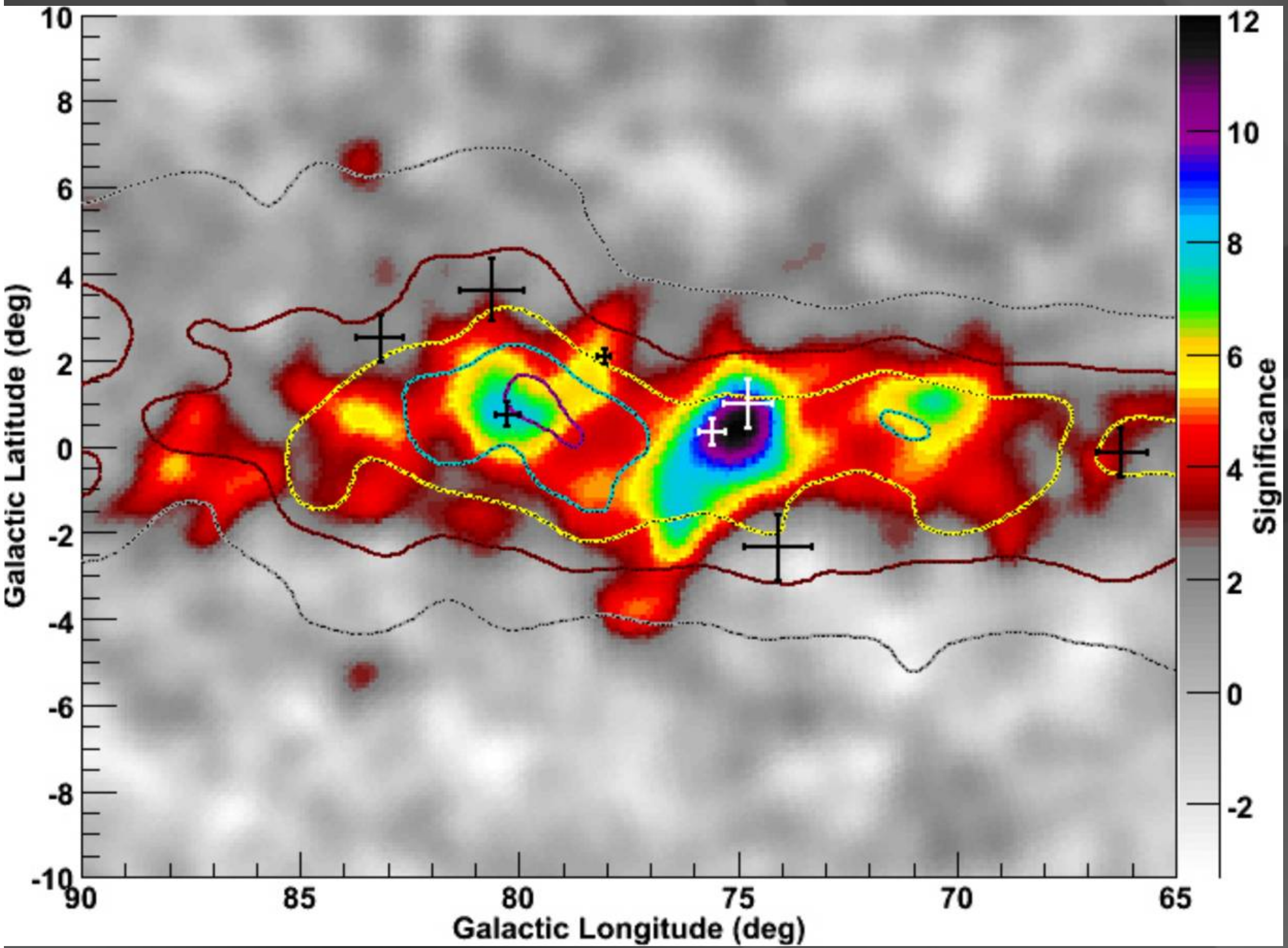
$$\text{number of events} = \text{Area Time} \int dE\, \frac{dN_\nu}{dE} P_{\nu\to\mu}$$

$$= 1.5 \ln\left(\frac{E_{\max}}{E_{\min}}\right) \text{events per km}^2 \text{per year per source!}$$

reject background

$\to E \geq 40\, TeV$

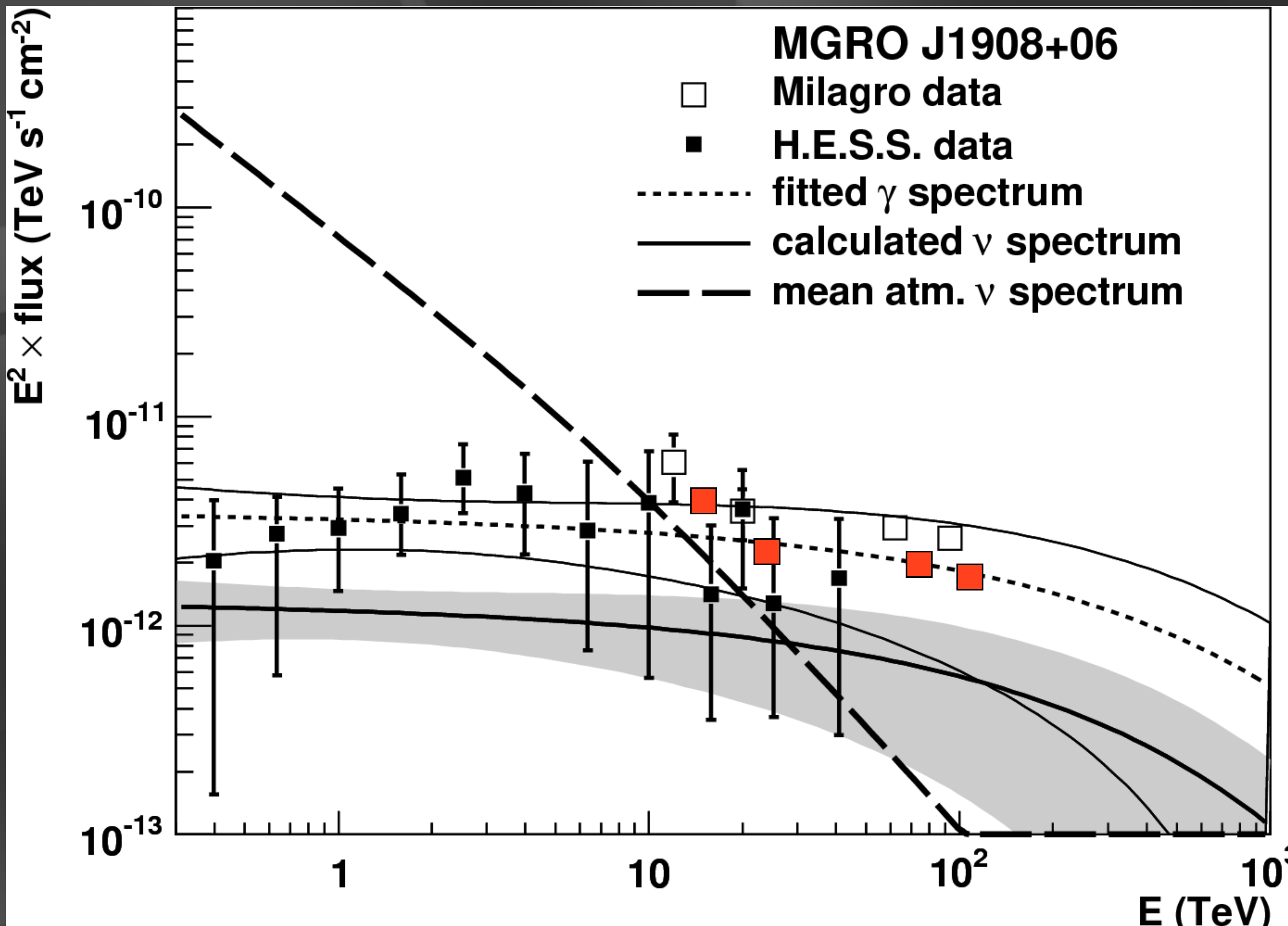
Cygnus region : Milagro



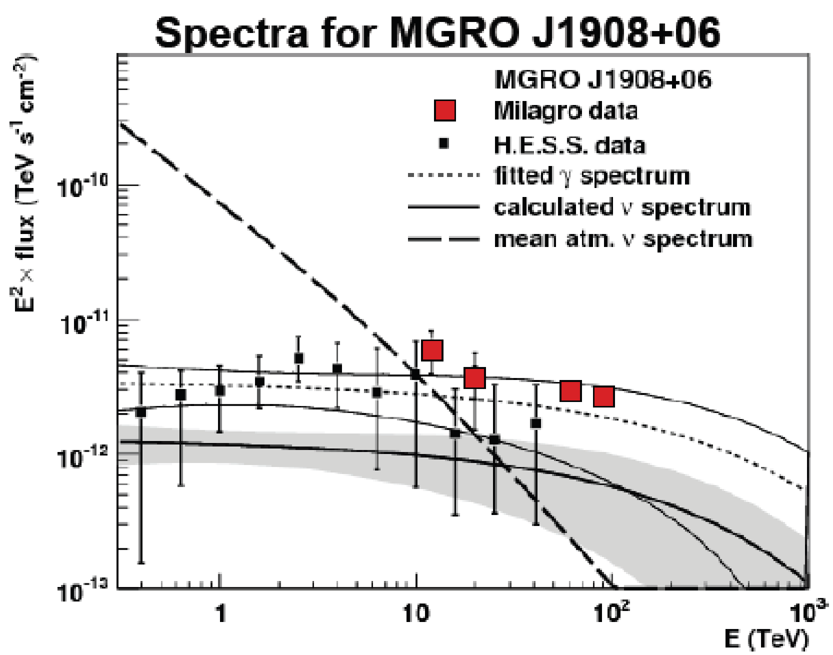
direct translation of TeV gamma rays into TeV neutrinos:

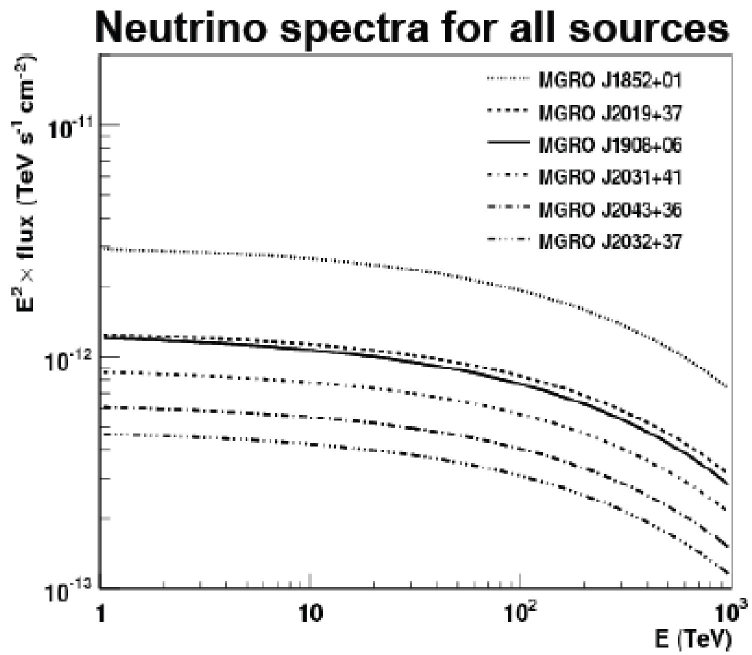
3 ± 1 ν per year in IceCube per source

MGRO J1908+06: the first Pevatron?



Gamma and Neutrino Spectra





Halzen, Kappes, O'Murchadha: arXiv:0803.0314

- Assumed E^{-2} with Milagro normaliztion (MGRO J1908 index = 2.1)
- ν spectrum cutoff @ 180 TeV

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equal energy in cosmic rays and neutrinos

$$\rho_{\nu+\bar{\nu}}(E) = \frac{E}{E_p}\left[\xi_z t_H\right]\left[c\dot{\rho}_p\right]$$

$$\rho_{\nu+\bar{\nu}}(E) = 4\pi E^2 \frac{dN_\nu}{dE}$$

$$\dot{\rho}_p(E_p) = E_p^2 \frac{d\dot{N}_p}{dE_p} \approx 10^{44}\,\mathrm{erg\,Mpc^{-3}\,yr^{-1}}$$

$$\xi_z t_H = \text{evolution of sources} \times \text{Hubble time}$$

$$\Rightarrow E^2 \frac{dN_\nu}{dE} \approx 10^{-11}\,\mathrm{TeV\,cm^{-2}\,s^{-1}\,sr^{-1}}$$

equal energy in cosmic rays and neutrinos

actually…

$$\rho_{\nu+\bar{\nu}}(E) = n_{\text{int}} \frac{E}{E_p}\left[\xi_z t_H\right]\left[c\dot{\rho}_{cr}\right]$$

- $n_{\text{int}} \le 1$ transparent (to photons) source; equality is the WB bound
- $n_{\text{int}} \ge 1$ obscured source
- observed flux is well below the WB bound (at $20 \sim 100\,\text{PeV}$); have to observe PeV photons

flux of extragalactic cosmic rays

ankle → one 10^{19} eV particle per km squared per year per sr

$$E^2 \frac{dN}{dE} = \frac{10^{19}\, eV}{(10^{10}\, cm^2)(3\times 10^7 \sec)\ sr}$$

$$= 3\times 10^{-11}\, TeV\, cm^{-2} \sec^{-1} sr^{-1}$$

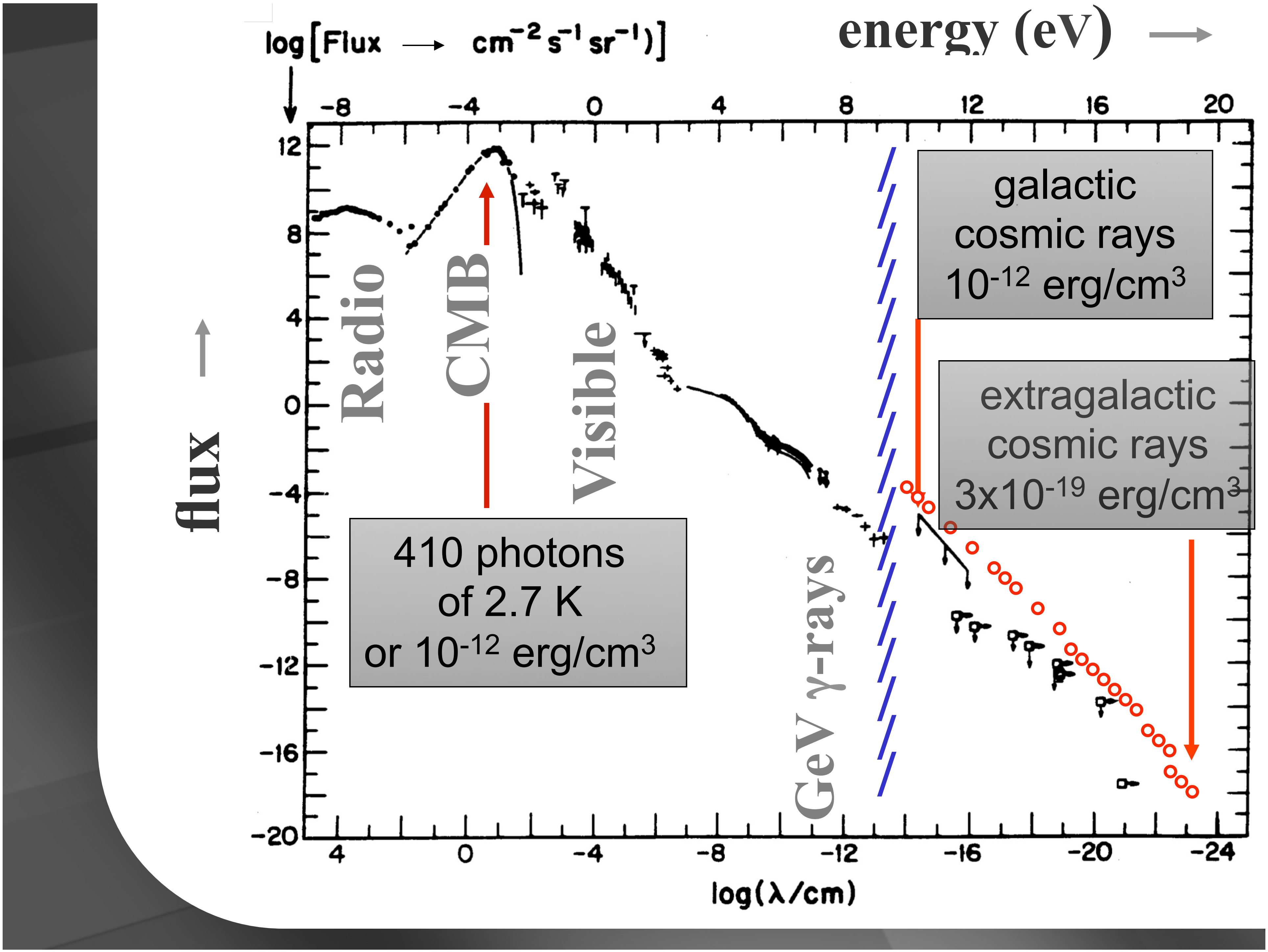
total flux = velocity x density

$$\boxed{4\pi\int dE\left(E\frac{dN}{dE}\right) = c\,\rho_E}$$

$$\rho_E = \frac{4\pi}{c}\int \frac{3\times 10^{-11}}{E}\,dE\;\frac{TeV}{cm^3}$$

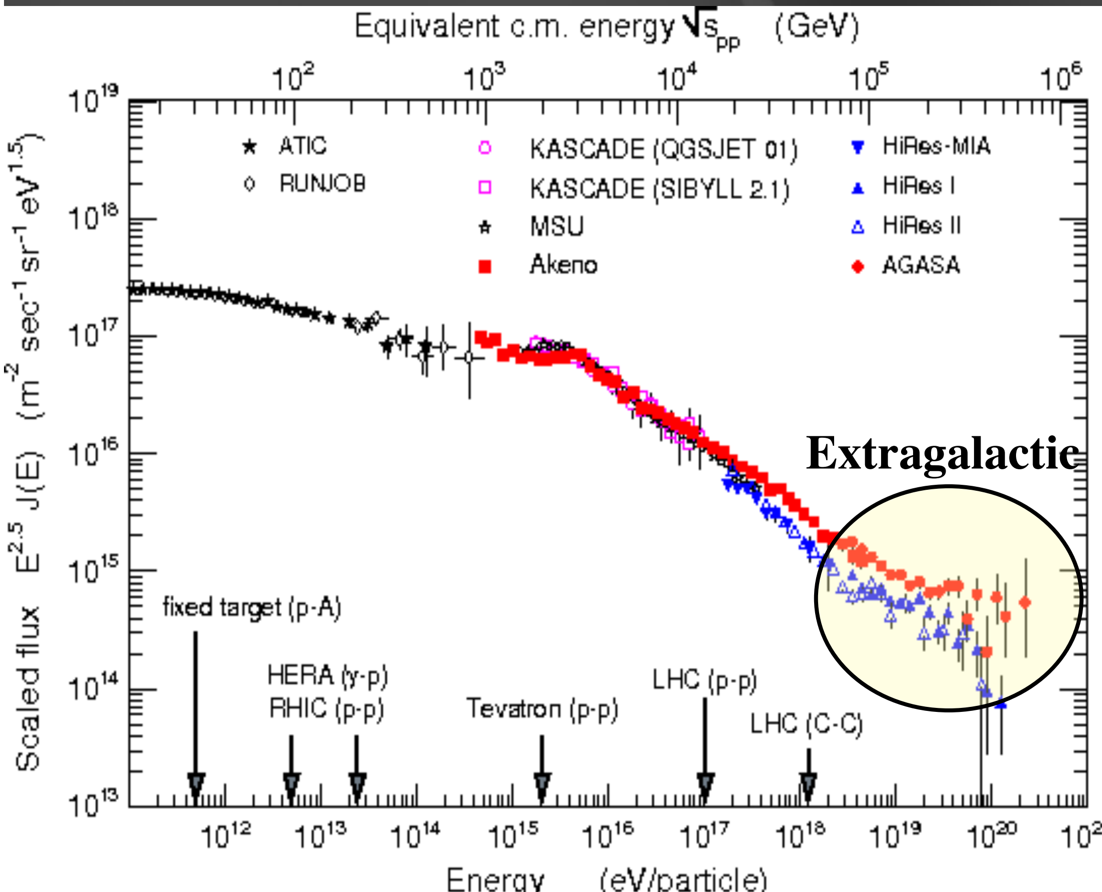
$$= \cdots \log\frac{E_{\max}}{E_{\min}} \cong 10^{-19}\;\frac{TeV}{cm^3}$$

$1\,TeV \cong 1.6\,erg$



energy (eV)
log[Flux → cm-2 s-1 sr-1)]
-8 -4 0 4 8 12 16 20
12 8 4 0 -4 -8 -12 -16 -20
4 0 -4 -8 -12 -16 -20 -24
log(λ/cm)
flux
Radio
CMB
Visible
GeV γ-rays
galactic cosmic rays 10-12 erg/cm3
extragalactic cosmic rays 3x10-19 erg/cm3
410 photons of 2.7 K or 10-12 erg/cm3

Cosmic Rays & GRBs



observed energy density of extragalactic CR:

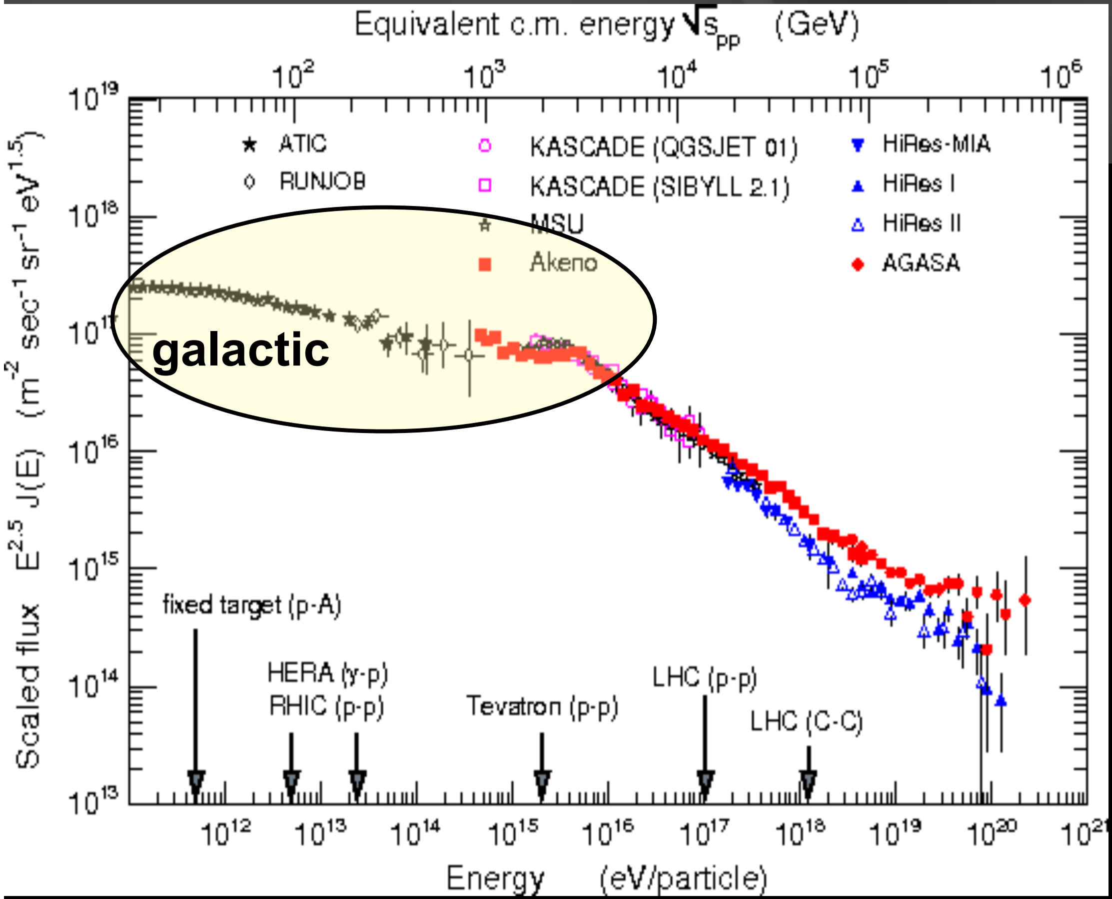
~ 10^{-19} erg / cm^3

Gamma-Ray Bursts:
2×10^{51} ergs x 300/Gpc^3 x 10^{10} yr

~ 10^{-19} erg / cm^3

GRBs provide environment and energy to explain the extragalactic cosmic rays!

Cosmic Rays & SNRs



observed energy density of galactic CR:

$\sim 10^{-12}$ erg/cm^3

supernova remnants:
10^{50} ergs every 30 years

$\sim 10^{-12}$ erg/cm^3

for steady state of CR with lifetime $\sim 10^6$ years

SNRs provide the environment and energy to explain the galactic cosmic rays!

300 GRB per Gigaparsec^3 per year for 10^{10} years (Hubble time)

$$2\times10^{52}\,erg\times\frac{300}{Gpc^3\;yr}\times10^{10}\,yr = 3\times10^{-19}\,\frac{erg}{cm^3}$$

- correct cosmology: same answer
- Fermi: photon (electron) energy less than this ?

$$1Gpc^3 = 2.9\times10^{82}\,cm^3 \qquad Hubble\;time = 10^{10}\;years$$

→ energy in extra-galactic cosmic rays is ~ 3×10^{-19} erg/cm^3

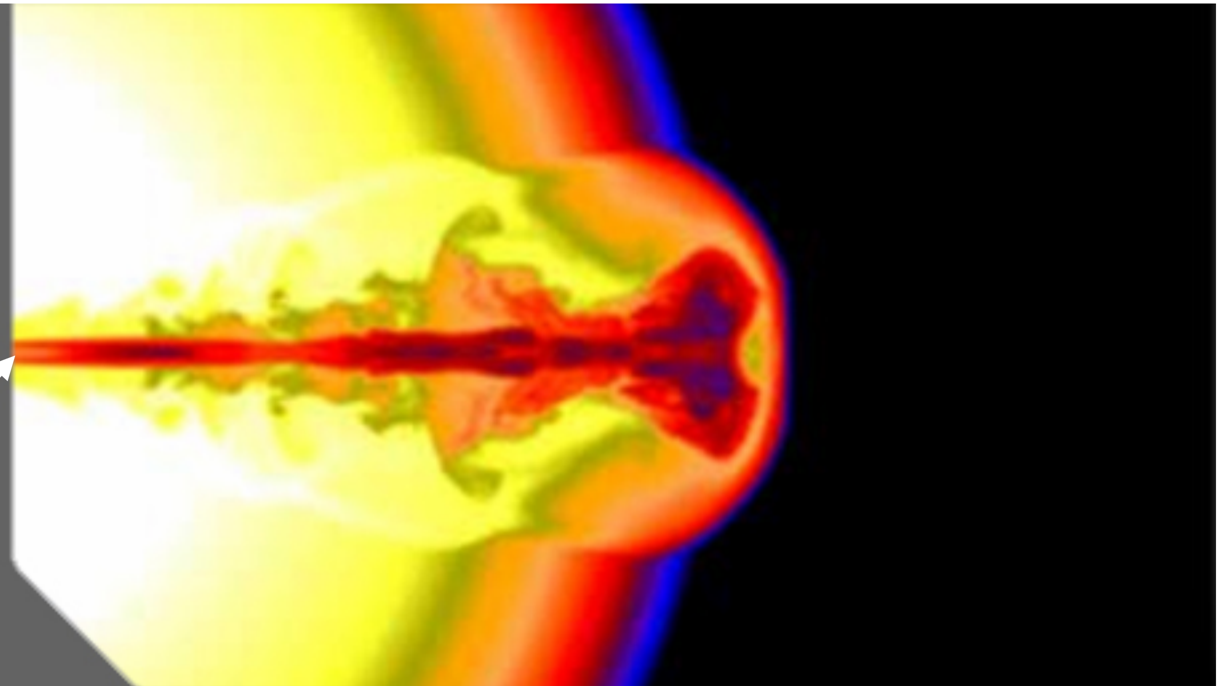
3×10^{44} erg/s per active galaxy

2×10^{51} erg per gamma ray burst

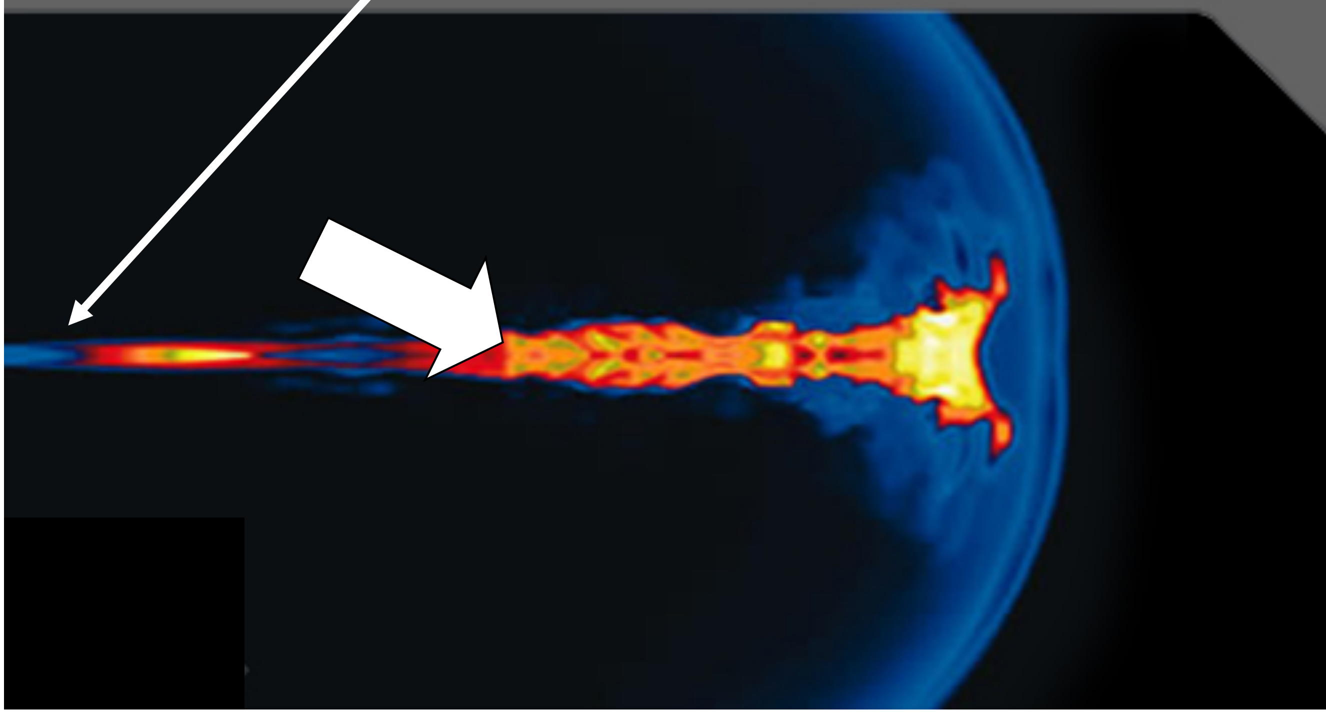
energy in cosmic rays ~ photons

collapse of massive star produces a

gamma ray burst



spinning black hole



shocks produced in the outflow of the spinning black hole: electrons (and protons ?)

speed of light
size of grb ?

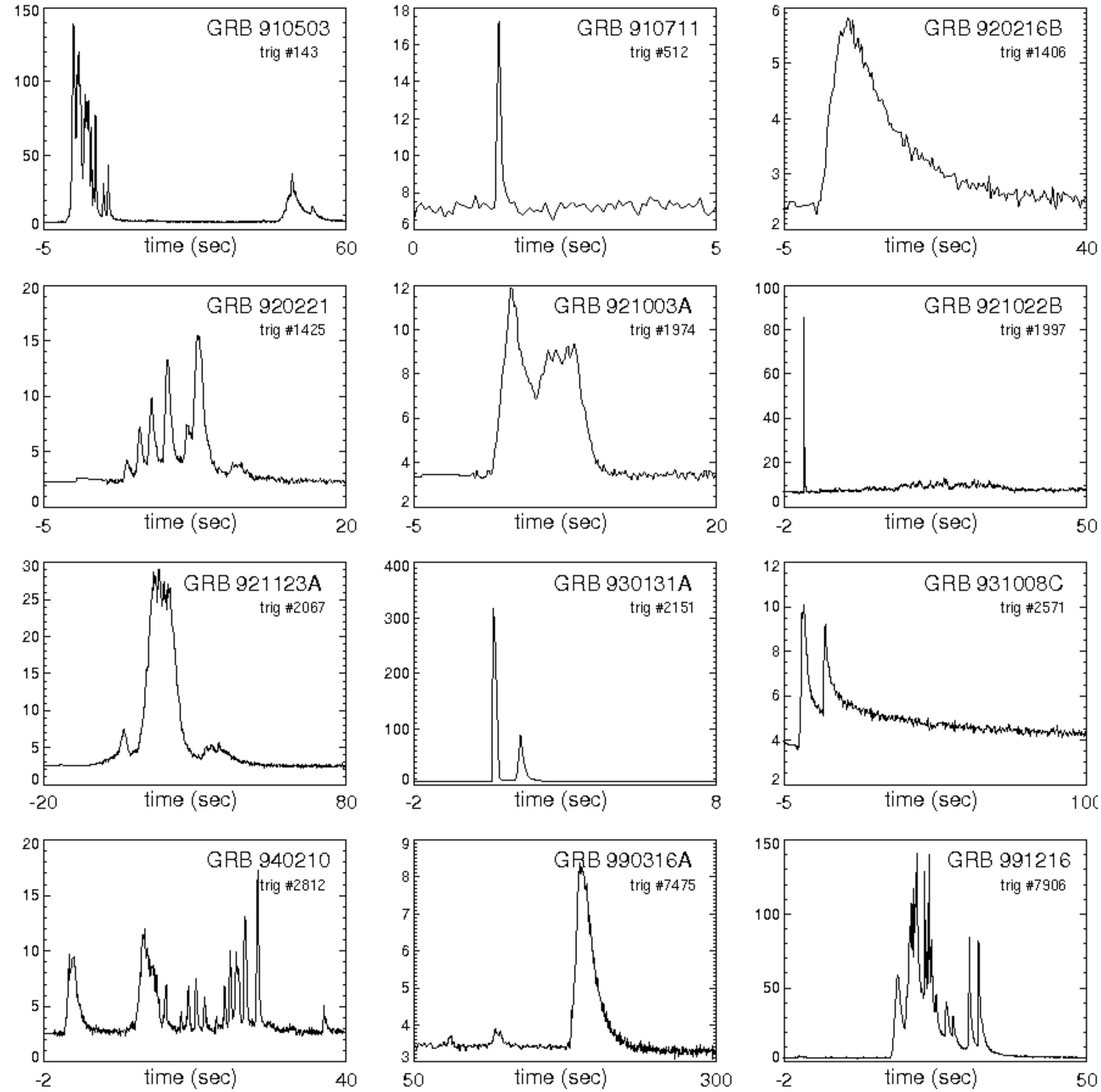
characteristic
time Δt ?

→1-10 msec

(not duration!)

size of the
accelerator

$\Delta R < c\,\Delta t$
~ 100 km



GRB: γ =300 → 10^{20}eV

- $acceleration\ time \le duration\ of\ the\ burst$

$$\frac{r_{gyro}}{c} \le \frac{1}{c}\frac{R}{\gamma} \quad with \quad r_{gyro} = \frac{1}{\gamma}\frac{E}{eB}$$

$$B > 10T \left[\frac{E}{10^{20}eV}\right]\left[\frac{10^{11}m}{R}\right]$$

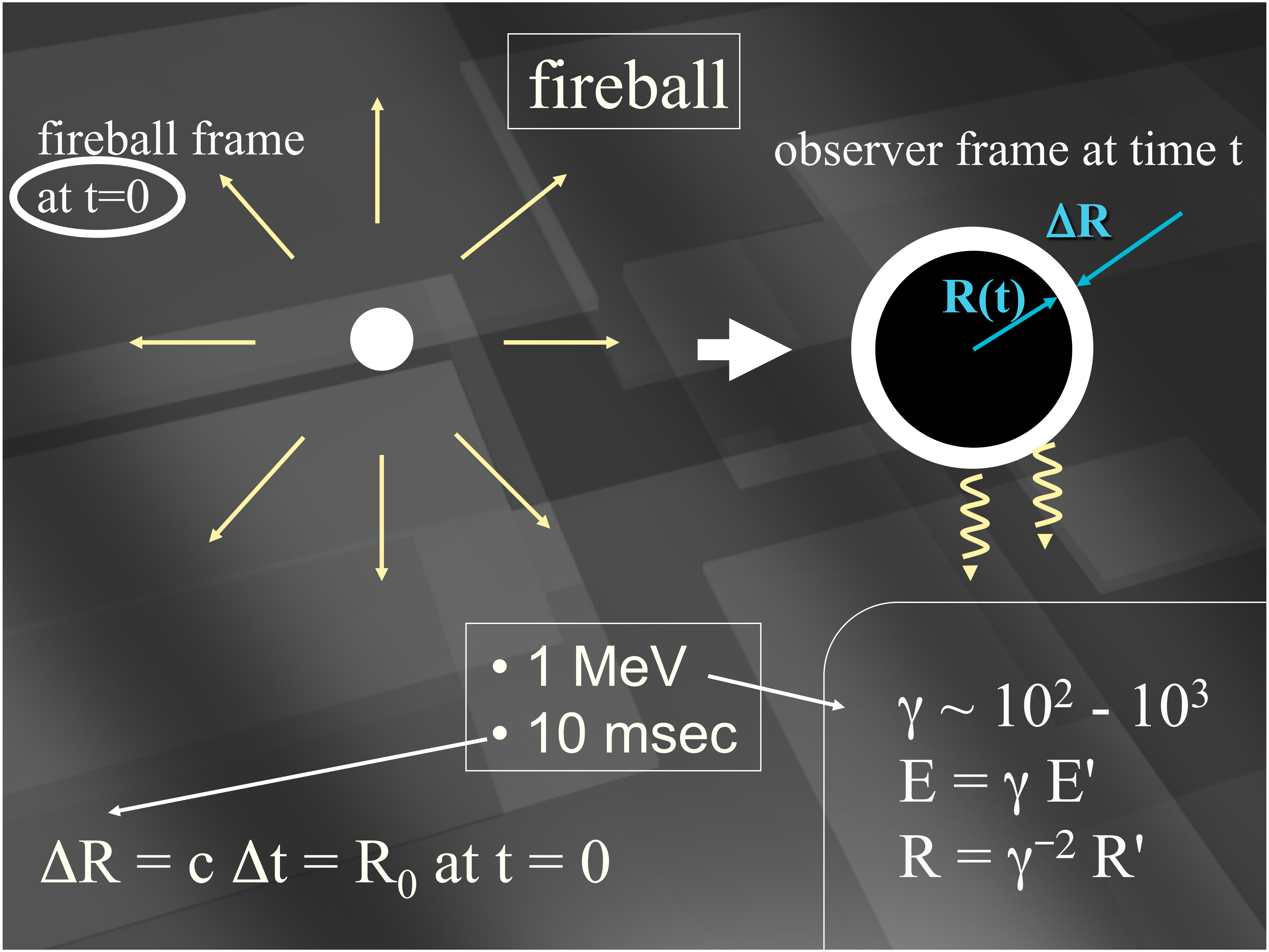
- $electron\ synchrotron\ losses \le energy\ gained\ in\ acceleration$

$$\frac{t_{syn}}{c} \ge \frac{1}{c}\frac{1}{\gamma}\frac{E}{eB} \quad with \quad t_{syn} = \frac{1}{c}\frac{1}{\gamma_p n_e \sigma_T} \quad in\ restframe\ n_e = \frac{\left(m_e c^2\right)^2 B^2}{6\pi\, m\Delta\Delta_p^3} \Leftarrow \frac{B^2}{8\pi} = \frac{L}{\gamma^2}$$

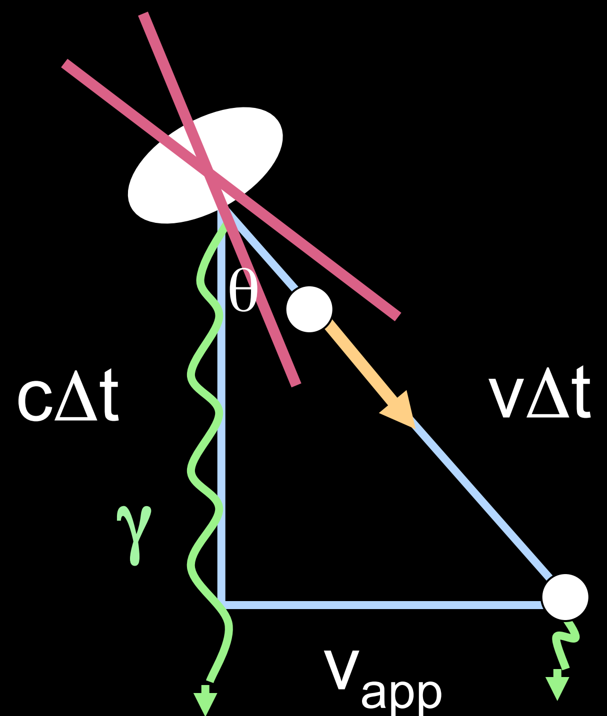
$$B \le 10T \left[\frac{\gamma}{300}\right]^2 \left[\frac{10^{20}eV}{E}\right]^2$$

- $combine$

$$R \approx \gamma^2 c\Delta t \ge 10^{10} m \left[\frac{300}{\gamma}\right]^2 \left[\frac{E}{10^{20}eV}\right]^3 \quad \Rightarrow \quad \gamma \ge 130 \left[\frac{E}{10^{20}eV}\right]^{\frac{3}{4}} \left[\frac{0.01s}{\Delta t}\right]^{\frac{1}{4}}$$


fireball
fireball frame
at t=0
observer frame at time t
ΔR
R(t)
• 1 MeV
• 10 msec
$\Delta R = c\,\Delta t = R_0$ at $t = 0$
$\gamma \sim 10^2 - 10^3$
$E = \gamma E'$
$R = \gamma^{-2} R'$

superluminal motion: boosted accelerators



$\beta = v/c \qquad \gamma = (1-\beta^2)^{-1/2}$

$D^{-1} = (1+z)\,(1 - \beta\cos\theta)\,\gamma$

$$\boxed{\begin{aligned} E_{\mathrm{obs}} &= \gamma\, E' \\ \Delta t_{\mathrm{obs}} &= \gamma^{-1}\, \Delta t' \end{aligned}}$$

$$v_{app} = \frac{v\Delta t \sin\vartheta}{\dfrac{c\Delta t}{c} - \dfrac{v\Delta t\cos\vartheta}{c}}$$

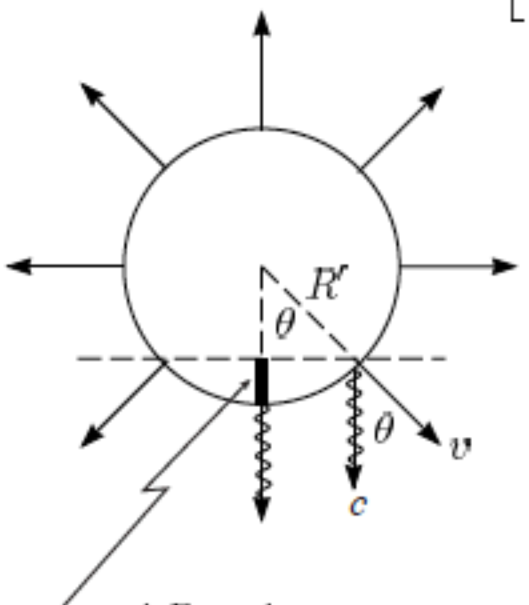
strongest effect :

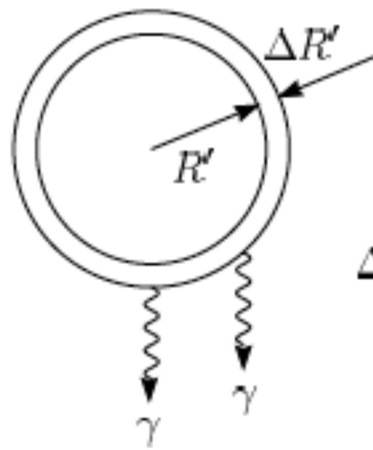
$$\frac{dv_{app}}{d\vartheta} = 0 \;or\; \cos\vartheta = \frac{v}{c} = \beta$$

$$or\;\; D = \gamma$$

$$R_0' \simeq 100 \text{ km} \qquad \cos\theta = \frac{v}{c}$$

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2} \simeq 10^2\text{–}10^3$$





$$\Delta R' = \gamma c \Delta t$$

$$R' = \gamma^2 c \Delta t$$

$$\gamma = 10^2\text{–}10^3$$

$$\Delta t = 1 \text{ msec}$$

$$\Delta t = \frac{\Delta R}{c} = \frac{1}{c}(R - R\cos\theta)$$

$$= \frac{R}{c}\left(1 - \frac{v}{c}\right) \simeq \frac{R}{2c}\left(1 - \frac{v^2}{c^2}\right)$$

$$\simeq \gamma^2 \frac{R'}{2c}$$

Simple relativistic kinematics (see Fig. 9) relates the radius and width R' and $\Delta R'$ to the observed duration of the photon burst $c\Delta t$:

$$\begin{aligned} R' &= \gamma^2 (c\Delta t) && (34) \\ \Delta R' &= \gamma c \Delta t && (35) \end{aligned}$$

From the observed GRB luminosity L_γ, we compute the photon energy density in the shell:

$$U'_\gamma = \frac{(L_\gamma \Delta t)/\gamma}{4\pi R'^2 \Delta R'} = \frac{L_\gamma}{4\pi R'^2 c \gamma^2} \tag{36}$$

The pion production by shocked protons in this photon field is, as before, calculated from the interaction length:

$$\frac{1}{\lambda_{p\gamma}} = N_\gamma \sigma_\Delta \langle x_{p\to\pi}\rangle = \frac{U'_\gamma}{E'_\gamma}\sigma_\Delta \langle x_{p\to\pi}\rangle \qquad \left(E'_\gamma = \frac{1}{\gamma}E_\gamma\right). \tag{37}$$

Also as before, σ_Δ is the cross section for $p\gamma \to \Delta \to n\pi^+$ and $\langle x_{p\to\pi}\rangle \simeq 0.2$. The fraction of energy going into π-production is

$$f_\pi \cong \frac{\Delta R'}{\lambda_{p\gamma}} \tag{38}$$

$$f_\pi \simeq \frac{1}{4\pi c^2}\frac{L_\gamma}{E_\gamma}\frac{1}{\gamma^4 \Delta t}\sigma_\Delta \langle x_{p\to\pi}\rangle \tag{39}$$

$$\begin{aligned} f_\pi \simeq\ & 0.14 \left[\frac{L_\gamma}{10^{51}\,\mathrm{ergs}^{-1}}\right]\left[\frac{1\ \mathrm{MeV}}{E_\gamma}\right]\left[\frac{300}{\gamma}\right]^4\left[\frac{1\ \mathrm{msec}}{\Delta t}\right] \\ & \times \left[\frac{\sigma_\Delta}{10^{-28}\,\mathrm{cm}^2}\right]\left[\frac{\langle x_{p\to\pi}\rangle}{0.2}\right]. \end{aligned} \tag{40}$$

$$E'_p = \frac{m_\Delta^2 - m_p^2}{4E'_\gamma}.$$

$$E_p = 1.4 \times 10^{16}\,\mathrm{eV}\left(\frac{\gamma}{300}\right)^2\left(\frac{1\ \mathrm{MeV}}{\mathrm{E}_\gamma}\right)$$

$$E_\nu = \frac{1}{4}\langle x_{p\to\pi}\rangle E_p \simeq 7\times 10^{14}\,\mathrm{eV}.$$

We are now ready to calculate the neutrino flux:

$$\frac{dN_\nu}{dE_\nu} = \frac{c}{4\pi}\frac{U'_\nu}{E'_\nu} = \frac{c}{4\pi}\frac{U_\nu}{E_\nu} = \frac{c}{4\pi}\frac{1}{E_\nu}\left[\frac{1}{2} f_\pi t_H \dot{E}\right], \tag{43}$$

where the factor 1/2 accounts for the fact that only 1/2 of the energy in charged pions is transferred to $\nu_\mu + \bar{\nu}_\mu$. As before, $\dot{E}$ is the injection rate in cosmic rays beyond the ankle ($\sim 4 \times 10^{44}\,\mathrm{erg\,Mpc^{-3}\,yr^{-1}}$) and t_H is the Hubble time of $\sim 10^{10}$ Gyr. Numerically,

$$\begin{aligned} \frac{dN_\nu}{dE_\nu} &= 2 \times 10^{-14}\,\mathrm{cm^{-2}\,s^{-1}\,sr^{-1}} \left[\frac{7 \times 10^{14}\,\mathrm{eV}}{\mathrm{E}_\nu}\right]\left[\frac{f_\pi}{0.125}\right]\left[\frac{t_H}{10\ \mathrm{Gyr}}\right] \\ &\quad \times \left[\frac{\dot{E}}{10^{44}\,\mathrm{erg\,Mpc^{-3}\,yr^{-1}}}\right] \end{aligned} \tag{44}$$

photon density in the fireball

$$n_\gamma = \frac{U'_\gamma}{E'_\gamma} = \frac{\dfrac{L_\gamma \Delta t/\gamma}{4\pi R'^2 \Delta R'}}{\dfrac{E_\gamma}{\gamma}}$$

$$R' = \gamma^2 c \Delta t$$

$$\Delta R' = \gamma c \Delta t$$

note: for $\gamma = 1$ (no fireball) the optical depth of photons in the fireball is →

$$\tau_{opt} = \frac{R_0}{\lambda_{Th}} = R_0 n_\gamma \sigma_{Th} \sim 10^{15} \text{ for } 10^{52} \text{ erg in } R_0 \sim 10 \text{ km}$$

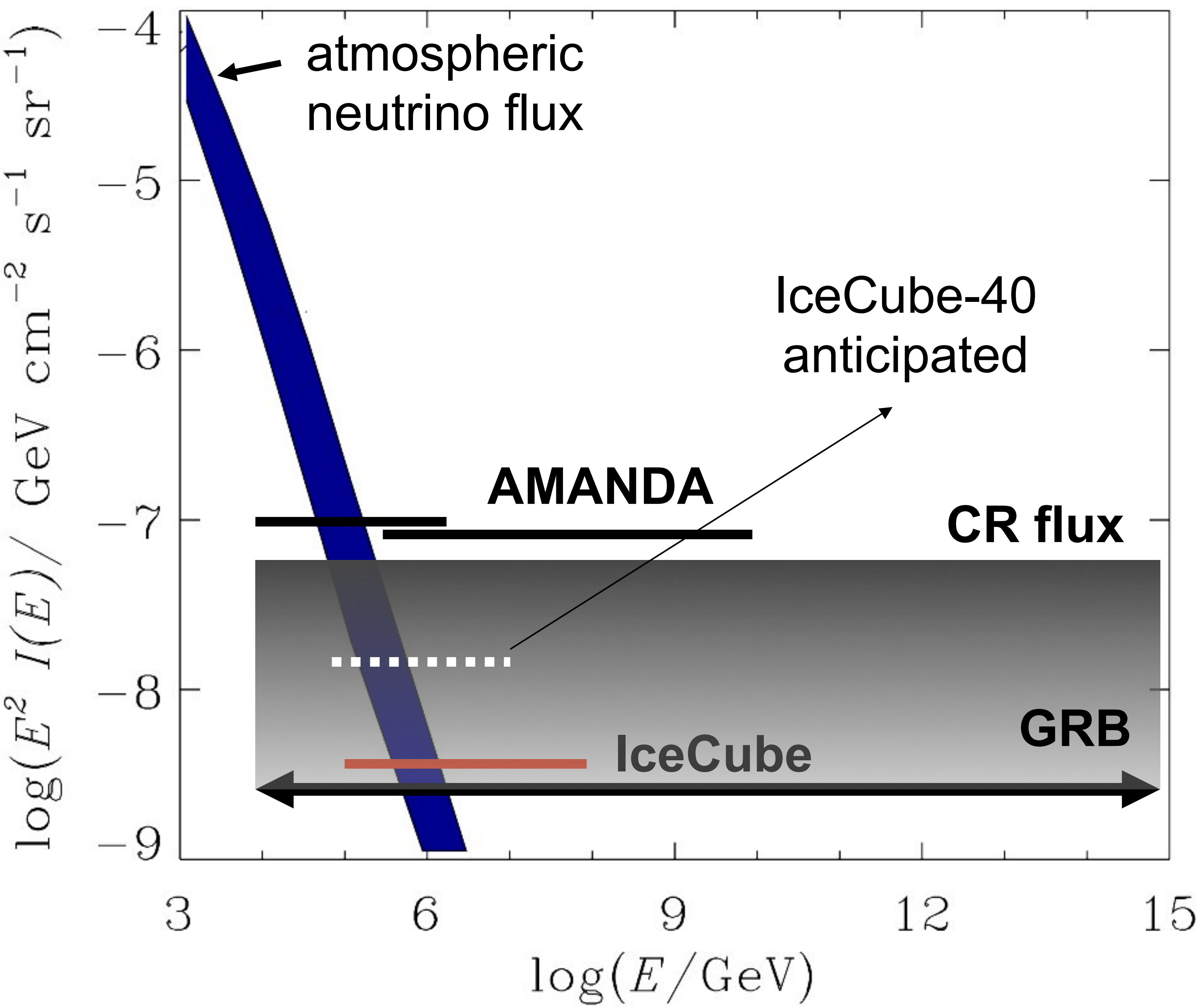
numerology

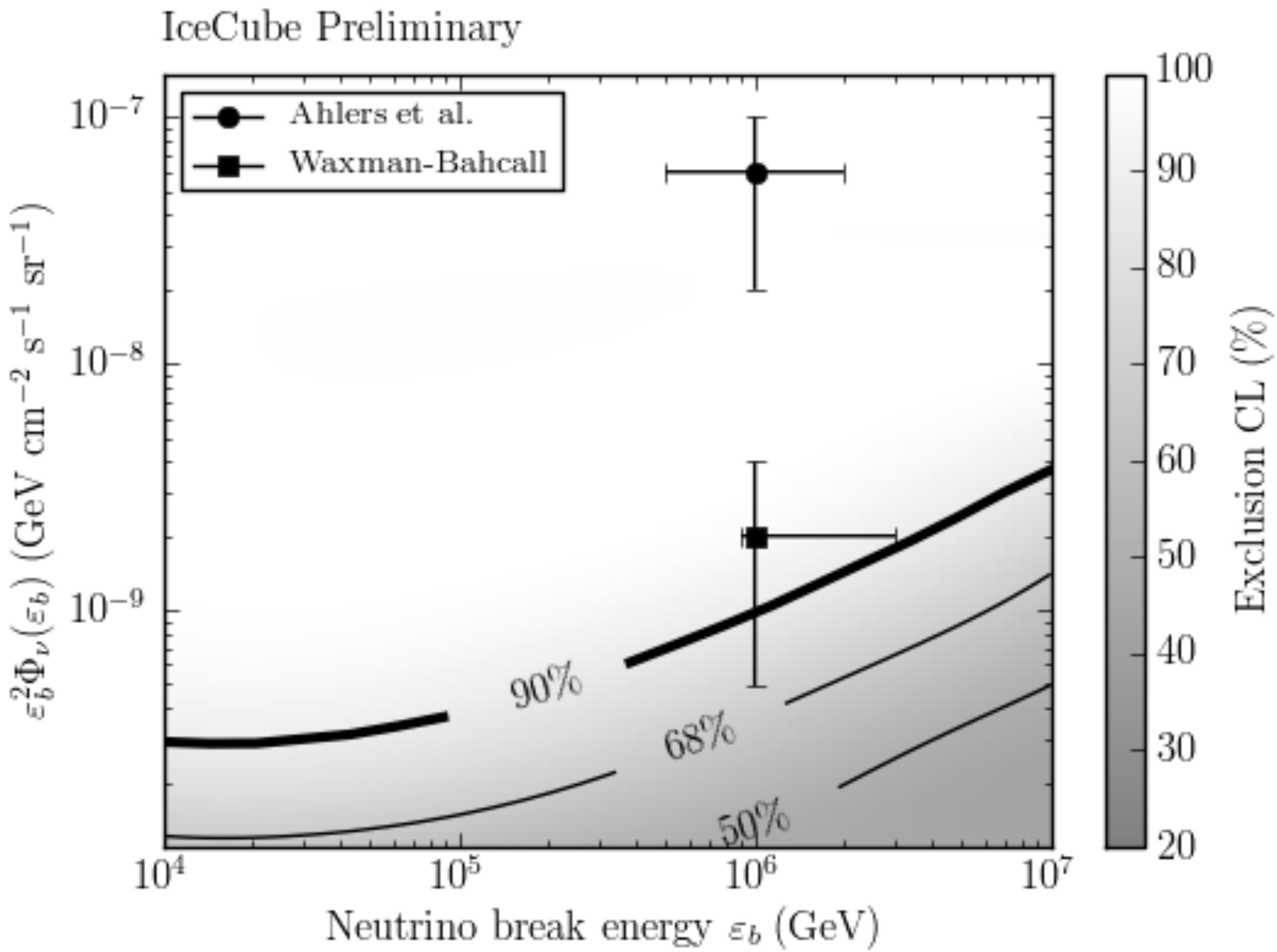
$L_\gamma = 10^{51}$--10^{52} erg/s
$R_0 = 100$ km ($\Delta t = 1$-10 msec)
$E_\gamma = 1$ MeV

$\gamma = 300$
$dE/dt = (1\text{-}4)\times10^{44}$ erg Mpc^{-3}yr^{-1}
$t_H = 10^{10}$ years

$P_{det} = 10^{-6} E_\nu^{0.8}$ (in TeV)
$\sigma_{p\gamma} = 10^{-28}$ cm^2 for $p+\gamma \rightarrow n+\pi$
$x_{p\rightarrow\pi} = f_\pi = 0.2$

neutrinos associated with extragalactic cosmic rays




IceCube Preliminary
Ahlers et al.
Waxman-Bahcall
90%
68%
50%
10^{-7}
10^{-8}
10^{-9}
10^4
10^5
10^6
10^7
Neutrino break energy ε_b (GeV)
$\varepsilon_b^2\Phi_\nu(\varepsilon_b)$ (GeV cm^{-2} s^{-1} sr^{-1})
100
90
80
70
60
50
40
30
20
Exclusion CL (%)

cosmic rays and gamma rays

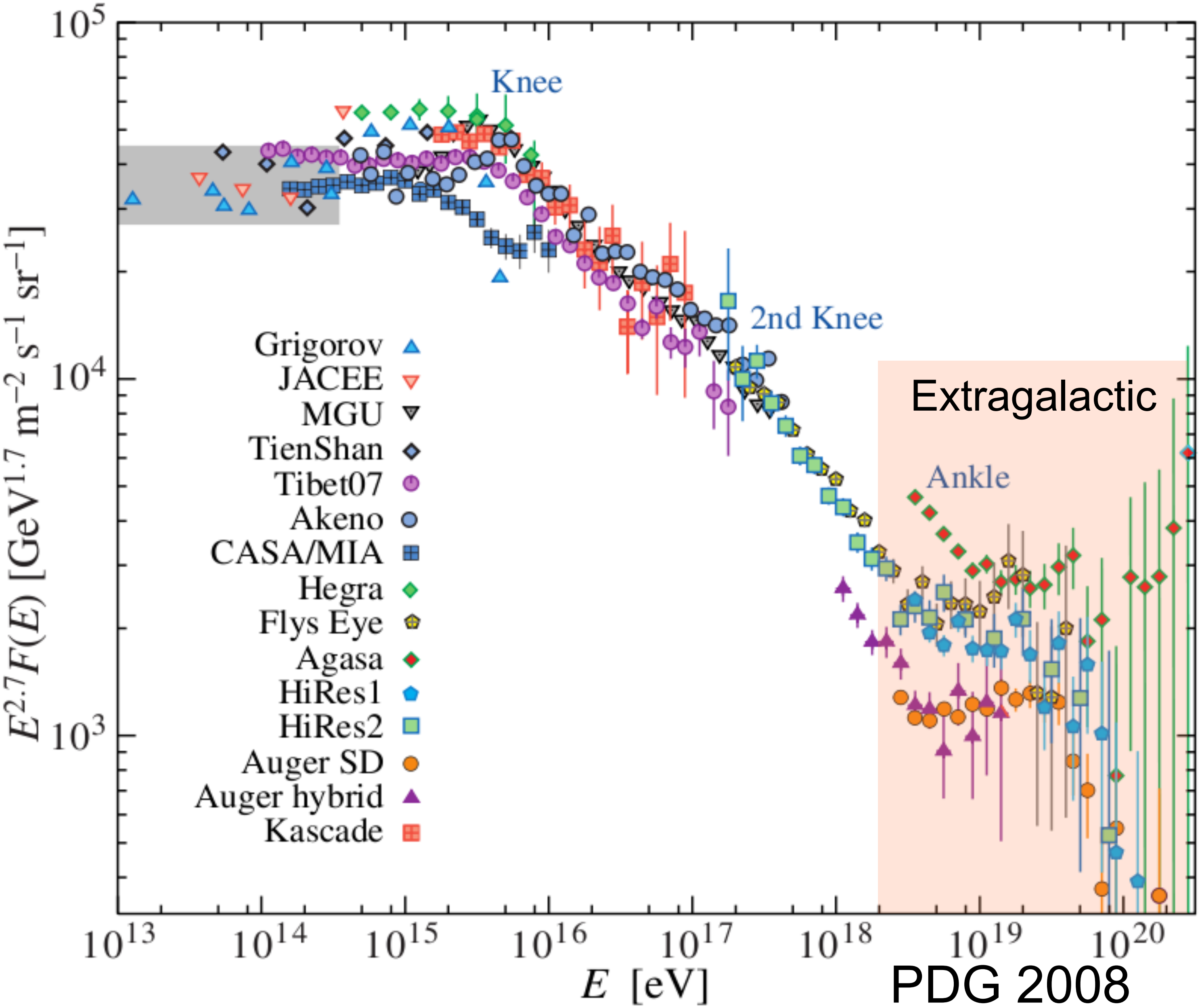
→ *Galactic cosmic rays*
• energetics of the sources
• supernova remnants: photons observed?

→ *extragalactic cosmic rays*
• energetics of the sources
• gamma ray bursts
• active galaxies

→ *general*
• shock acceleration
• an example: supernova remnants

strong shocks

cosmic rays



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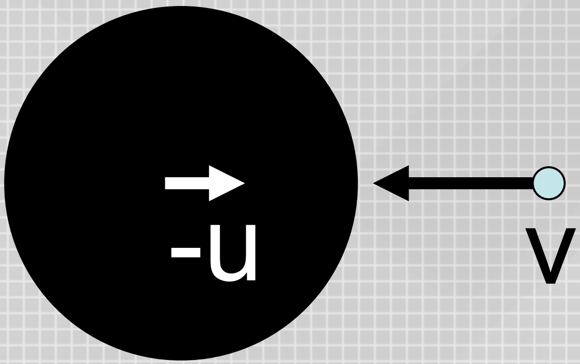
non-relativistic shock



$$\text{before} \quad \text{our frame} \qquad E_i = \frac{1}{2} m v^2$$

$$\text{before} \quad \text{cloud frame} \qquad E_i^c = \frac{1}{2} m (v+u)^2$$

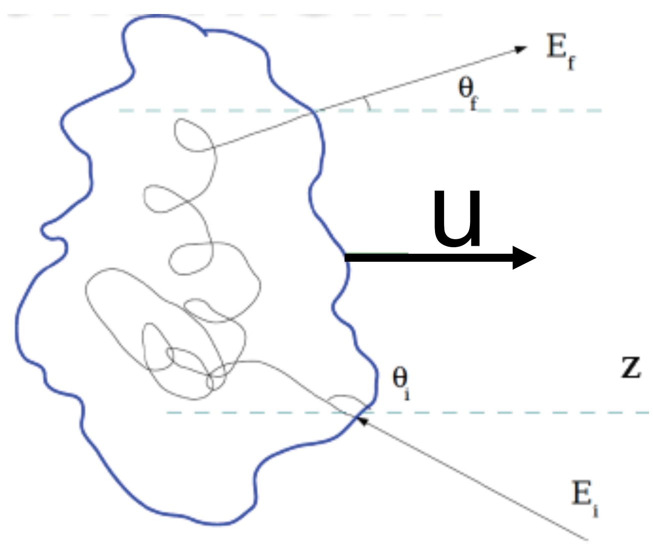
$$\text{after} \quad \text{cloud frame} \qquad E_f^c = \frac{1}{2} m (-v-u)^2$$

$$\text{after} \quad \text{our frame} \qquad E_f = \frac{1}{2} m (-v-2u)^2$$

$$\frac{E_f - E_i}{E_i} = \frac{\frac{1}{2} m (-v-2u)^2 - \frac{1}{2} m v^2}{\frac{1}{2} m v^2}$$

$$= \frac{4uv + 4u^2}{v^2} \approx 4\left(\frac{u}{c} + \frac{u^2}{c^2}\right)$$

before	*our frame*	E_i, p_i
before	*cloud frame*	$E_i^c = \gamma\left(E_i + c\beta\, p_i\right)$ *and* $p_i^c = \gamma\left(p_i + \beta\dfrac{E_i}{c}\right)$
after	*cloud frame*	$E_f^c = E_i^c$ *and* $p_f^c = -p_i^c$
after	*our frame*	$E_f = \gamma\left(E_f^c - c\beta\, p_f^c\right) = \gamma\left(E_i^c + c\beta\, p_i^c\right)$



$$with\ E_i \approx cp_i$$

$$E_f \approx E_i\gamma^2(1+\beta)^2$$

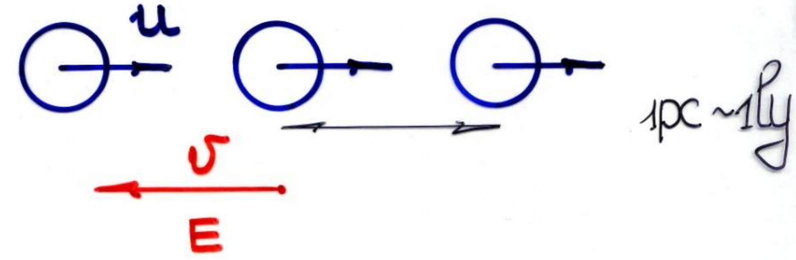
$$with\ \gamma^2 \approx 1+\beta^2+\ldots$$

$$E_f \approx E_i\left(1+2\beta+2\beta^2+\ldots\right)$$

$$\frac{E_f - E_i}{E_i} \approx 2\left(\beta+\beta^2\right)$$

acceleration time too long

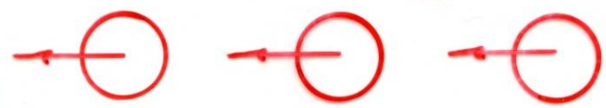
FERMI 49 : particle collides with stellar clouds



$$\frac{\Delta E}{E} \sim \frac{u}{v}\left(\frac{u}{c}+\frac{v}{c}\right) \qquad u \ll v \simeq c$$

$$\boxed{\frac{\Delta E}{E} \sim \frac{u}{c} + \left(\frac{u}{c}\right)^2}$$

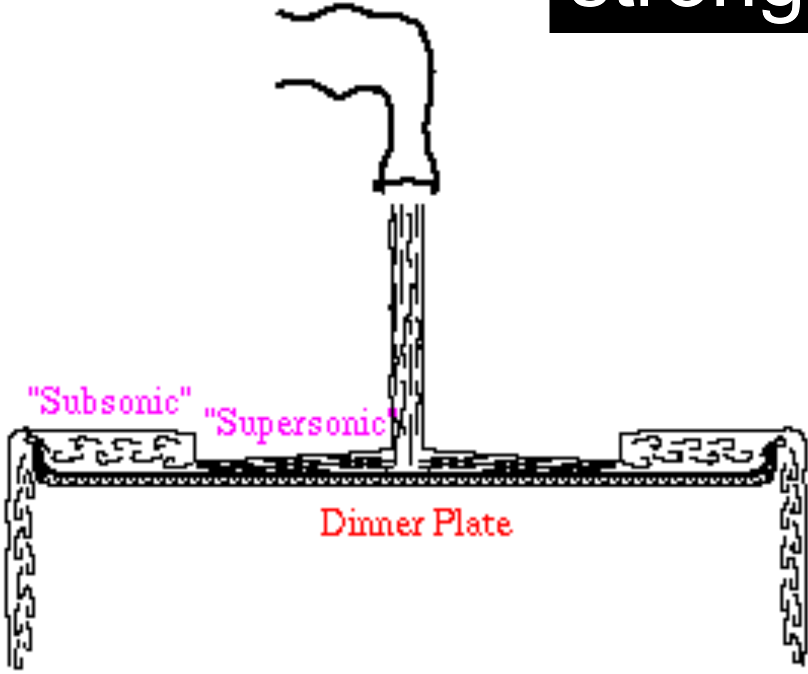
- some clouds ($\frac{v-u}{v+u}$ fraction, Doppler) come from the wrong direction

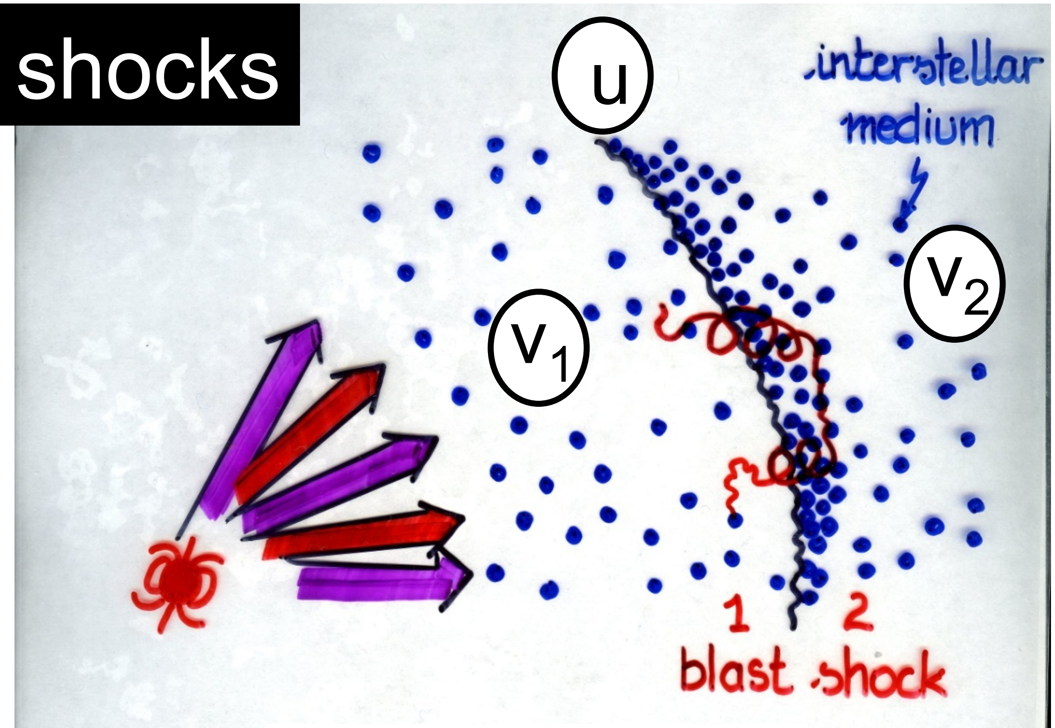


$$\frac{\Delta E}{E} \sim \frac{u}{v}\left(\frac{u}{c}+\frac{v}{c}\right)\left(\frac{u+v}{v}\right) - \frac{u}{v}\left(-\frac{u}{c}+\frac{v}{c}\right)\left(\frac{v-u}{v}\right)$$

- u/c term cancels

strong shocks





$$v_2 > v_1 \qquad \rho_2 < \rho_1$$

$$\frac{\Delta E}{E} \Leftrightarrow \frac{v_2 - v_1}{v_2}$$

shockspeed u > v_{sound}

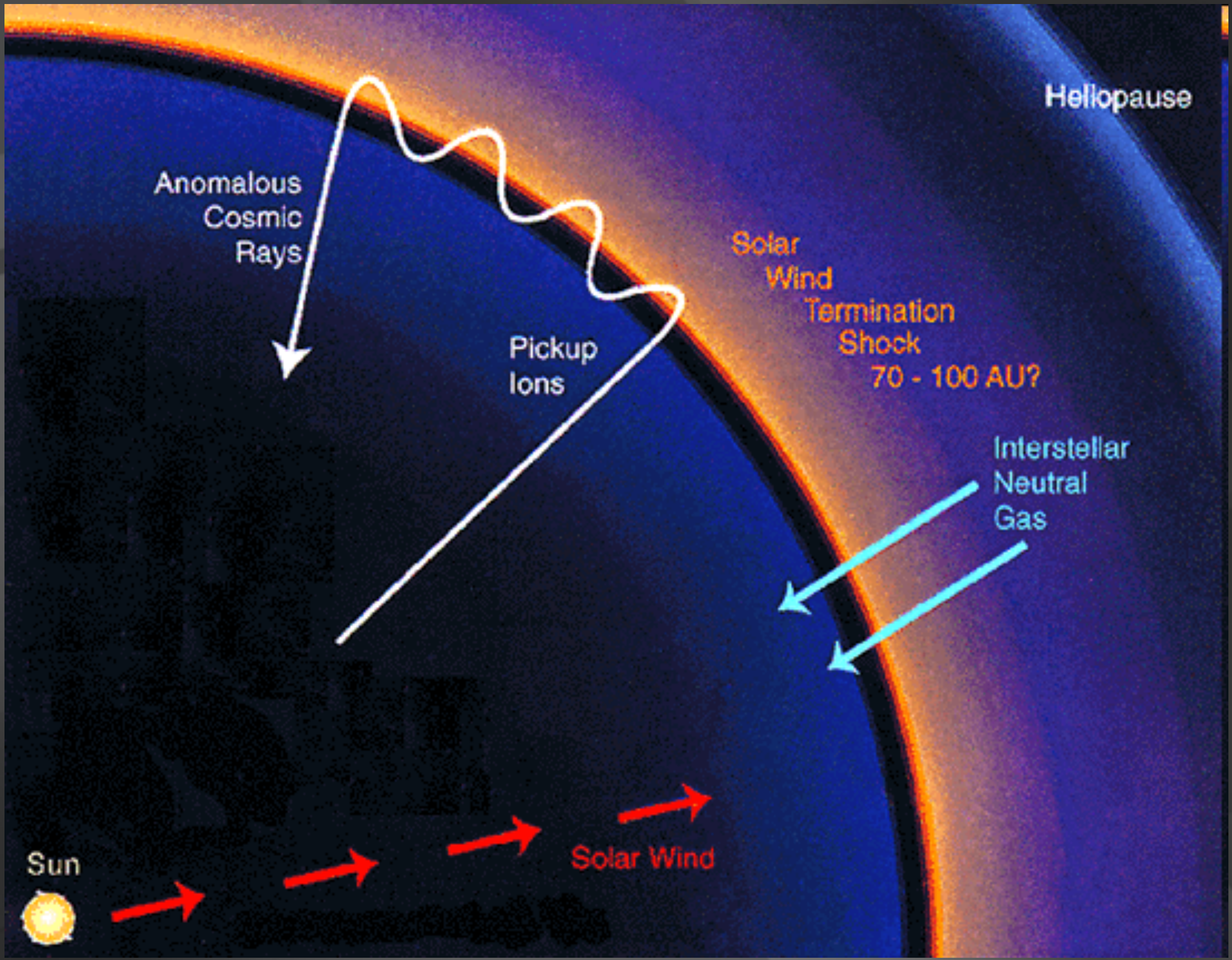
definition:

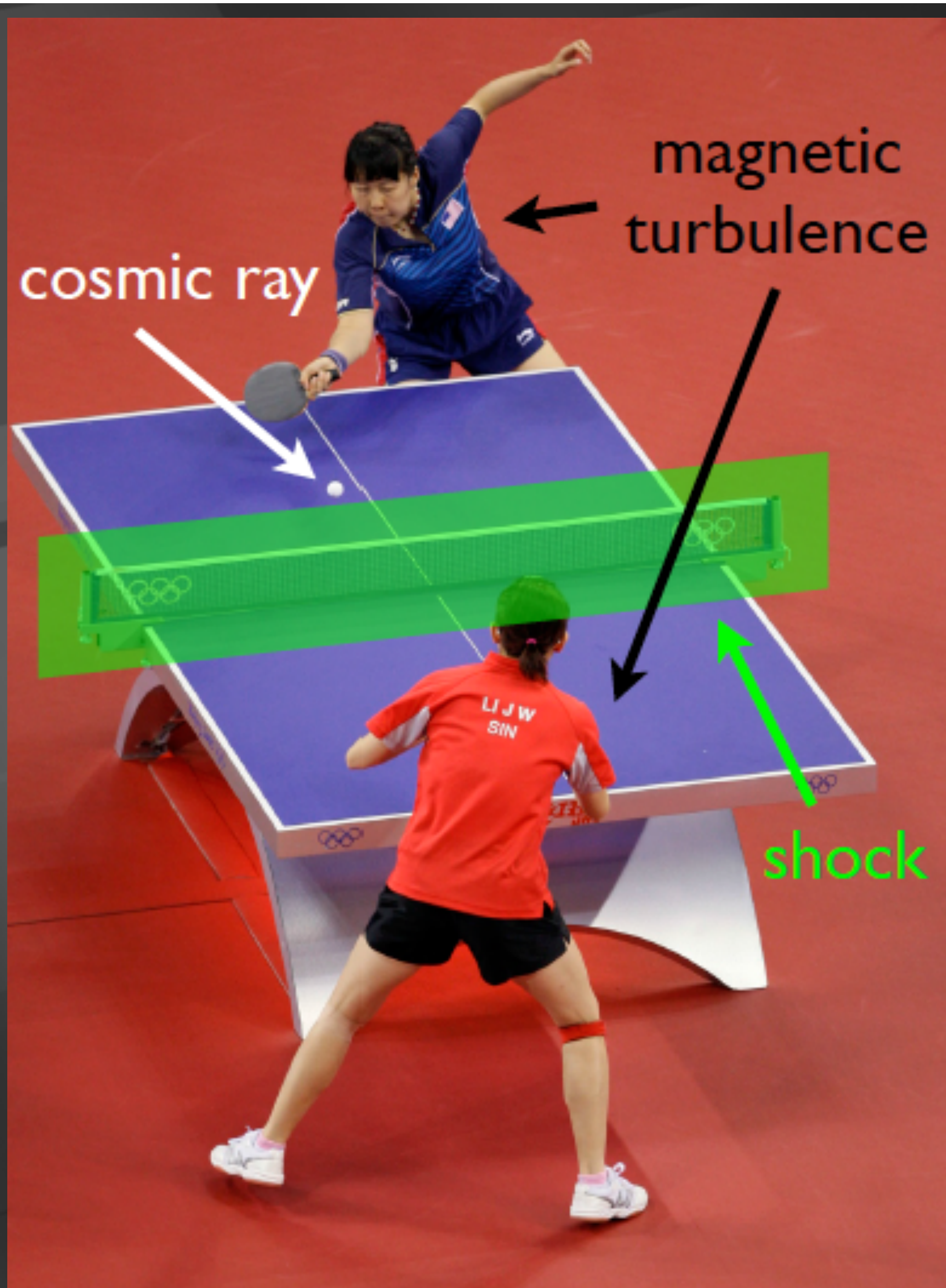
- v_2 upstream
- v_1 downstream

mass conservation in shock frame:

$$\rho_1 v_1 = \rho_2 v_2 \quad \Rightarrow \quad \frac{v_1}{v_2} = \frac{\rho_2}{\rho_1} = \frac{1}{4}$$

solar wind termination shock



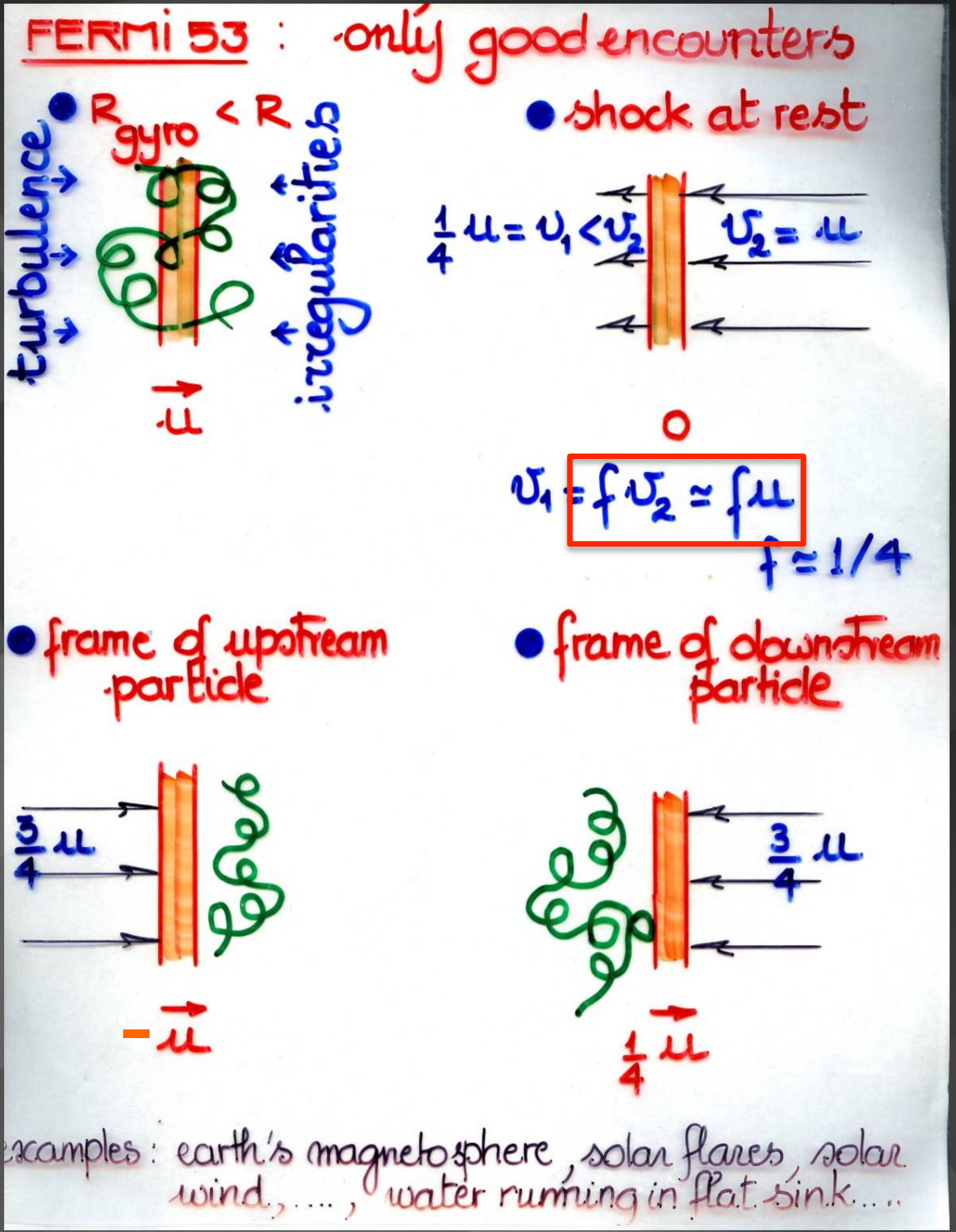

magnetic
turbulence
cosmic ray
LI J W
SIN
shock

only head-on collisions

mass conservation :

$$\rho_1 v_1 = \rho_2 v_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\rho_2}{\rho_1} = \frac{1}{4}$$



shock spectrum

$$\frac{dN}{dE} \sim \frac{1}{E^2}$$

$$\frac{\Delta E}{E} = \beta^k$$ energy gain/encounter

$$\frac{\Delta N}{N} = -P^k$$ probabilty to cross shock

$$\frac{\Delta N}{N} = -\left(\frac{\beta}{P}\right)^k \frac{\Delta E}{E}$$ by dividing the two

$$\int_{N_0}^{N} \frac{\Delta N}{N} = -\left(\frac{\beta}{P}\right)^k \int_{E_0}^{E} \frac{\Delta E}{E}$$ integrate

$$\log \frac{N}{N_0} = -\gamma \log \frac{E}{E_0}$$

$$N = N_0 \left(\frac{E}{E_0}\right)^{-\gamma} \text{ and } \frac{dN}{dE} \sim E^{-1-\left(\frac{\beta}{P}\right)^k} \sim E^{-2}$$

… and the answer is E^{-2}

$$\boxed{\ln\beta = \frac{u}{c} \quad and \quad \ln P = -\frac{u}{c}}$$

$$\beta = \frac{E_f}{E_i} = 1 + \frac{\Delta E}{E} = 1 + 2 \times \left(\frac{\mathrm{v}_{rel}}{c}\right) \times \cos\theta \Rightarrow 1 + \frac{u}{c}$$

roundtrip

angular integration #1

$$P = 1 - P_{esc} = 1 - \frac{\rho_{cr}\mathrm{v}_{up}}{\rho_{cr}\frac{c}{4}} = 1 - \frac{\rho_{cr}\frac{u}{4}}{\rho_{cr}\frac{c}{4}} = 1 - \frac{u}{c} \quad shock\,restframe$$

angular integration #2

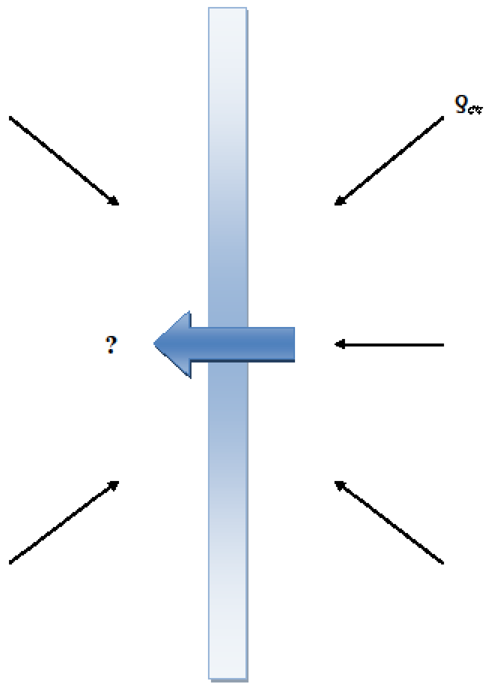
angular integral #1

$$\langle\frac{\Delta E}{E}\rangle = 2\frac{\mathrm{v}_{rel}}{c}\cos\theta \quad where\, \mathrm{v}\, is\, the\, relative\, velocity\ \frac{3}{4}u$$

$$average\, is\, over\, particle\, distribution\, N\left(\theta\right) = N\cos\theta$$

$$\langle\frac{\Delta E}{E}\rangle = 2\frac{\mathrm{v}_{rel}}{c}\frac{\int\cos\theta\, N\left(\theta\right)d\phi\, d\cos\theta}{\int N\left(\theta\right)d\phi\, d\cos\theta} = 2\frac{\mathrm{v}_{rel}}{c}\frac{\int_0^{\frac{\pi}{2}}\cos^2\theta\, d\cos\theta}{\int_0^{\frac{\pi}{2}}\cos\,\theta\, d\cos\theta} = \frac{4}{3}\frac{\mathrm{v}_{rel}}{c} = \frac{u}{c}$$

angular integral #2



$$flux\ F\ swept\ up\ from\ isotropic\ flux\quad \rho_{cr}$$

$$F = \int_{v>0} f(v)v\,d^3v = \pi\int_0^{\inf} f(v)v^3dv$$

$$\langle v\rangle = \frac{1}{n}\int f(v)v^3dv = \frac{4\pi}{n}\int_0^{\inf} f(v)v^3dv$$

$$combine$$

$$F = \frac{n\langle v\rangle}{4} \approx \frac{nc}{4}$$

maximum energy

$$\frac{dE}{dt} = f_{cross}\Delta E = \frac{c}{\lambda_{cross}}\beta E = \frac{\beta E}{t_{cross}} \le \beta E \frac{u}{r_{gyro}}$$

$$from \quad ut_{cross} \approx \lambda_{cross} \ge r_{gyro} \quad or$$

acceleration requires that the mean free path for magnetic scattering is larger than the gyroradius

maximum energy, ctd

$$\frac{dE}{dt} \le \beta E \frac{u}{r_{gyro}} = \left(\frac{u}{c}\right) E u \left(\frac{E}{ZeB}\right)^{-1} = \frac{u}{c} ZeBu$$

$$\boxed{E_{\max} = \frac{u}{c} ZeB(ut)}$$

the real problem: acceleration time

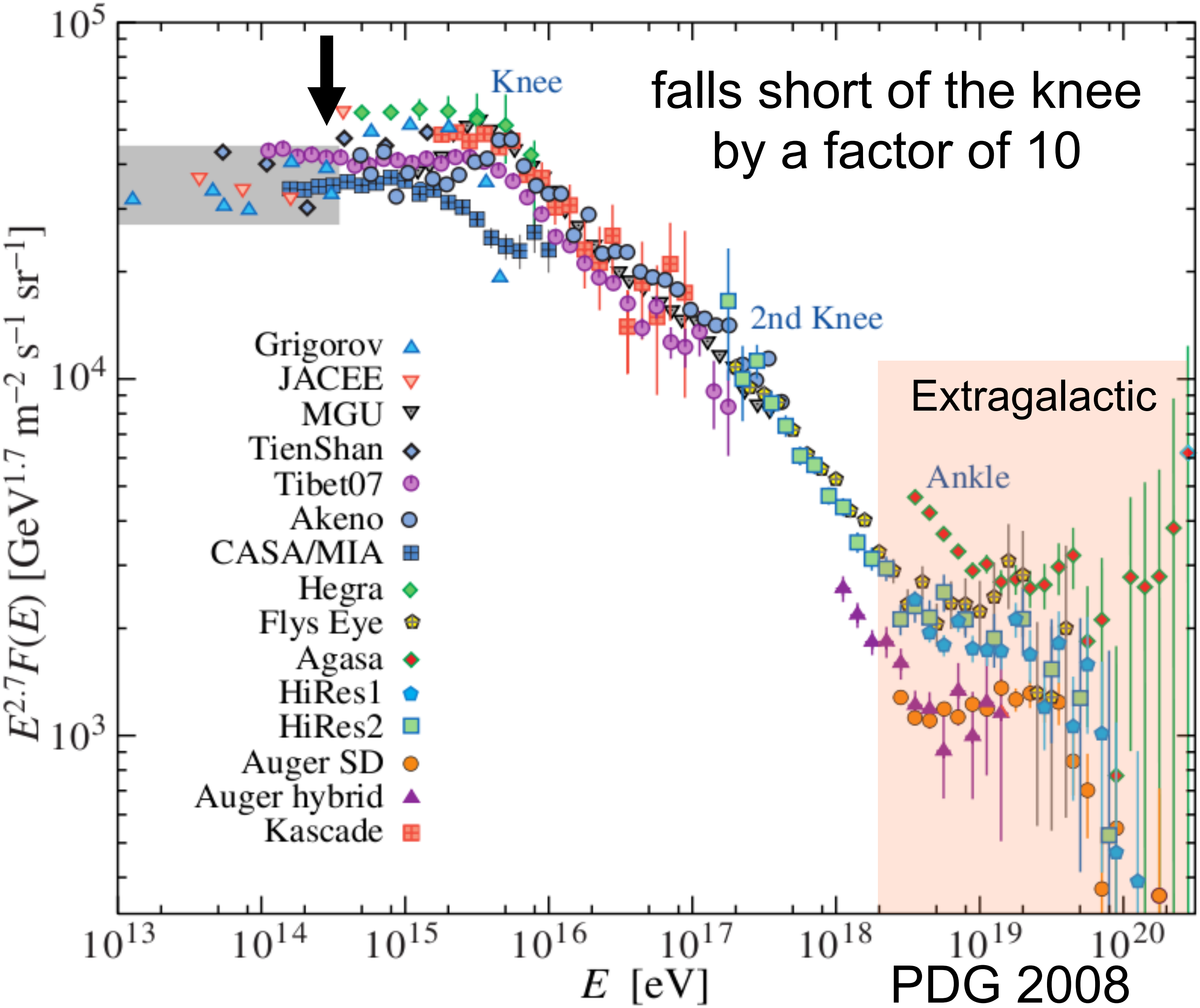
acceleration ends when the shock is diluted to a density similar that of the interstellar medium

$$\frac{4}{3}\pi(ut)^3\rho_{ism} \approx M_{sr}$$

$$E_{\max} = efficiency \times ZeB(ut) \leq Z\left(2.4\times10^5\ GeV\right)$$

for a remnant of 10 solar masses and shock speed u of (0.01-0.1) c

cosmic rays



PDG 2008

My favorite textbooks in Astroparticle

- T.K. Gaisser Cosmic Rays and Particle Physics (new edition 2016 available!)
- T. Stanev, High Energy Cosmic Rays, Springer, 2004
- M. S. Longair, High Energy Astrophysics, Cambridge U. Press, 2010
- D. Perkins, Particle Astrophysics (2nd ed. 2009)
- S. Rosswog & M.Bruggen High-Energy Astrophysics
- C. Grupen, Astroparticle Physics, Springer, 2005
- M. Spurio Particles and Astrophysics, Springer
- L. Bergstrom and A. Goobar, Cosmology and Particle A ger (2nd edition,2
- M. Fukugita and Y. Yanagida, Physics of Neutrinos, Spr